CALCULATION OF SPARES REQUIRED TO SUPPORT AIRCRAFT DEPLOYMENTS

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ABSTRACT

This report provides graphical methods that can be used by aircraft maintenance personnel to calculate the number of spares needed to support a deployment. It explains the theory involved, presents illustrative computations and shows how the required data can be obtained from the Aircraft Maintenance Management Information System (AMMIS). A user's manual is included to facilitate implementation.

The concepts are applicable to any equipment maintenance situation where repair is by replacement and repair of failed items is deferred. The concepts could be implemented in the air, land, sea and communications and electronics environments.

RESUME

Ce rapport présente une méthode graphique que le personnel chargé de la maintenance aéronef peut utiliser pour calculer le nombre de pièces de rechange nécessaire à supporter un déploiement. Il décrit la théorie utilisée, donne des exemplaires des calculs et indique comment extraire les données nécessaires du Système d'Information de Gestion-Maintenance Aéronefs (SDGMA). A titre de faciliter l'usage, on trouvera ci-inclus un guide d'emploi.

Les concepts sont de nature à s'appliquer à toutes situations de maintenance d'équipement où la réparation se fait par remplacement et, la réparation des itèmes en panne est différée. Les concepts peuvent être exécutés aussi dans l'environnement air, terre, mer, et communications et électroniques.

FOREWORD

This report documents work done by Lt. A. Rietmuller of the Directorate of Aerospace Support Engineering (DAS Eng) and Maj. B.F. Hough and Mr. I.W. Taylor of the Directorate of Logistics Analysis (D Log A).

ACKNOWLEDGEMENT

This report is intended to be of practical benefit to Canadian Forces aircraft squadrons. To the extent that this objective is achieved, a large measure of the credit must go to the many organizations and individuals who assisted with the project. In particular, the authors wish to acknowledge the assistance provided by:

a. Mr. G.R. Laing, of Datacap Ltd, who, working under contract with DAS Eng 5, wrote the data extraction and analysis computer programs;

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TABLE OF CONTENTS

	Page
ABSTRACT	(i)
RESUME	(i)
FOREWORD	(ii)
ACKNOWLEDGEMENT	(ii)
PART I - INTRODUCTION	1
Background	1
Aim	2
Data Sources	3
Selection of Items	3
Assumptions	4
Structure of the Report	5
PART II - APPLICATION OF POISSON THEORY	6
The Poisson Distribution	6
Mean Number of Equipment Replacements	8
Probability of Having Enough Spares	9
Example Problem - Poisson Distribution	13
A Compound Poisson Distribution	17
Example Problem - Compound Poisson	21
Data Extraction Technique	31
A Practical Example - CF116 Pack-Up Kit	33
Choosing Alternative Target Probabilities	38
PART III - CONSIDERATIONS WITH ACTUAL DATA	39
Visual Inspection of Histograms and Scattergrams	39
Trends in Equipment Reliability	40
Cyclic Data	44
Outliers	46
Data From Deployments	49
Summary	49

TABLE OF CONTENTS (cont'd)

	Page
PART IV - RELAXATION OF ASSUMPTIONS.....	50
Replacement of Component Parts Versus Complete Assemblies.....	50
Repair of LRUs During the Deployment.....	52
Taking Cost Into Account.....	52
Initial Provisioning of Repairables.....	54
War Reserve Stocks.....	56
PART V - PUTTING THESE IDEAS TO USE.....	58
REFERENCES.....	63
ANNEX A - INTERPRETATION OF POISSON THEORY.....	A-1
ANNEX B - DATA EXTRACTION.....	B-1
ANNEX C - PROPOSED USER'S INSTRUCTIONS.....	C-1
ANNEX D - CREATION OF THE NOMOGRAPHS.....	D-1

CALCULATION OF SPARES REQUIRED TO SUPPORT AIRCRAFT DEPLOYMENTS

PART I - INTRODUCTION

BACKGROUND

1. In December 1981, the Directorate of Logistics Analysis (D Log A) was asked to provide a method that could be used by aircraft maintenance personnel to calculate the number of aircraft spare parts required to support a deployment.

2. The need for such methodology was explained in Reference A:

> All squadrons with deployment requirements (Aurora excepted) have pack-up kits at present. The kit contents, in terms of items to be taken and their quantity, have been determined at base level by obtaining advice from Non-Commissioned Officers (NCOs). Rarely are calculations done; rather, the NCOs rely on their experience and judgement. This approach has shortcomings in that it is completely dependent on the experience and current background of the NCOs or Warrant Officers. The decisions are entirely subjective. No estimate is provided of the impact on operations of the deployed unit or home base 'making do' with less than the needed quantity of spares. These shortcomings could be overcome if the experience and judgement were augmented with tested and proven quantitative methods. Given that the calculations would be done at base level, a methodology that requires either a computer for implementation or specialists to interpret results is not appropriate. The method must be predicated on:

a. calculations easily done using nomographs (which are precisely calibrated curves) or programmable pocket calculators with the necessary software; and

b. results easily interpreted and verified by maintenance officers regardless of experience level.

3. The applicable methodology (for the most simple case) has been available for many years, a nomograph for the Poisson distribution having been published by the US Navy in 1964 in Reference B and later reprinted in Reference C. However, this methodology is not well known among Canadian Forces aircraft maintenance personnel. Also, to the best of our knowledge, nomographs for the more general case involving a Compound Poisson distribution have not been published. This is significant because the latter distribution may be more applicable than the Poisson to some aircraft pack-up kit calculations. Therefore, a set of nomographs was created for a Compound Poisson distribution. Finally, procedures for extracting required data from the Aircraft Maintenance Management Information System (AMMIS) were developed and documented. The data extraction technique:

a. distinguished equipment repair by replacement from repair without replacement; and

b. took into account fluctuations in the flying rate.

AIM

4. The aim of this report is to provide a graphical method that can be used to calculate the number of spares needed to support an aircraft deployment.

5. The theory will be explained, the procedures for obtaining the required data will be outlined and illustrative computations will be presented.

6. The concepts are applicable in any maintenance situation where repair is by replacement and repair of failed items is deferred. Therefore, the methodology may also be of use in the land, sea and communications and electronics environments.

DATA SOURCES

7. 434 Squadron provided an operational scenario (Ref.D) and a list of pack-up kit items for the CF116 Freedom Fighter aircraft deployments (Ref.E). Later, information was also obtained regarding the CP140 Aurora aircraft operations at CFB Greenwood and CFB Comox, through Director Maritime Aircraft Engineering and Maintenance. Equipment replacement data were obtained from the Aircraft Maintenance Management Information System (AMMIS). Flying hours data were obtained from the Aircraft Utilization Statistical Report (AUSR).

SELECTION OF ITEMS

8. Selection of items for inclusion in the 434 Squadron pack-up kit was based on criteria that the parts:

a. had a relatively high failure rate;

b. were critical to mission success;

c. were critical to flight safety;

d. were capable of being replaced in the field; and

e. were not too heavy or bulky to transport.

9. Should advice be needed regarding which items to include in a pack-up kit, a computer print-out can be obtained from DAS Eng 5 listing equipment by Work Unit Code in order of highest to lowest failure rate. Items could be selected from this list using the above criteria. Also, items essential for flight safety or critical to the mission should be included, even if they have a low failure rate.

10. How to decide 'which' items to consider for inclusion in the pack-up kit is not discussed further; the authors believe that the best approach is to rely on the expertise of technicians. Rather, this report focuses on the methodology for determining the 'quantity' of each pack-up kit line item required to support a specified aircraft deployment.

ASSUMPTIONS

11. It is assumed that:

a. repair is by replacement for items in the pack-up kit;

b. any given item in the pack-up kit is not a sub-assembly of another item in the kit;

c. the expected number of equipment replacements that will occur is a function of the duration of the deployment expressed in flying hours; and

d. AMMIS data is representative of deployments.

12. Also, it is assumed that when an operational unit is planning a deployment, the cost of spare parts is not a factor because the existing inventory has already been purchased. Rarely is there sufficient time, prior to a deployment, to increase the inventory through additional procurement. In

addition, it is assumed that weight and volume have been taken into account by technical experts during the selection of items to be included in the pack-up kit. Therefore, these factors will not be considered for deployment planning purposes. For initial provisioning of repairables or procurement of war reserve stocks, cost must be considered. Therefore, the applicable methodology for these cases is discussed briefly towards the end of this report.

13. Assumptions a. through d. are re-examined in Parts IV and V of the report and the effect of relaxing the assumptions or using alternative approaches is discussed.

STRUCTURE OF THE REPORT

14. In order to ensure an understanding of how the computations are performed, a series of example problems are presented and solved in Part II of this report. Each example introduces an additional consideration, with the final example being representative of actual deployment planning.

15. In Part III advice is given regarding resolution of problems experienced with 'real-life' data. In Part IV, some of the assumptions are relaxed and the resulting changes in methodology are discussed. In Part V the content of the report is briefly reviewed and potential users are encouraged to implement the concepts.

16. Annex A provides a more detailed interpretation of Poisson theory and Annex B explains how the data were extracted from AMMIS. The report also includes proposed user instructions (Annex C); these were written in the form of a manual for NCOs and junior officers which could be published as a Canadian Forces Technical Order. Finally, Annex D explains how the nomographs were created.

PART II - APPLICATION OF POISSON THEORY

17. In this part of the report, the Poisson theory is described and example computations are given.

THE POISSON DISTRIBUTION

18. In the simplest case, the Poisson distribution is used to calculate the required number of spare parts. It is applied by obtaining equipment replacement data from aircraft maintenance records and calculating the mean number (m) of equipment replacements per sample interval and the variance (v) about this mean. The Poisson distribution is applicable if the variance to mean ratio is one. It may also be applied when this ratio is less than one. However, if this ratio is significantly greater than one, a Compound Poisson distribution should be applied, as discussed later in the report.

19. The words mean and average are synonymous. Variance is a measure of the scatter or dispersion in the data obtained by calculating the average of the squared deviations from the mean. These terms are defined mathematically as follows:

$$m = \sum_{i=1}^{b} q_i / b \qquad \text{.....Equation \#1}$$

$$v = \sum_{i=1}^{b} (q_i - m)^2 / (b-1) \qquad \text{.....Equation \#2}$$

where: b = the number of observations;

q_i = the quantity in observation i; and

$\sum_{i=1}^{b}$ = is a mathematical notation meaning sum over each successive observation.

For further explanation see any good text-book on statistics. Reference F is recommended.

20. The equation for the Poisson distribution is given in Annex A. An equivalent form of the Poisson formula which can be used for step-by-step computation is:

$$P = \sum_{x=0}^{S} \frac{e^{-M}(M)^x}{x!} \qquad \text{.....Equation \#3}$$

where: P = probability of having a spare part available when required (i.e. probability of no stock-out);

S = number of spare parts needed to achieve P;

x = the number of spare parts in a given term of the equation, with x ranging from zero to S in successive terms;

e = the constant 2.71828...;

M = the mean number of equipment replacements that are expected for a particular item during a specified time interval; and

! = is the factorial symbol, a mathematical notation meaning the product of the integers from x to 1 inclusive.

21. The term e-M is the reliability of the equipment, that is, the probability of no failures occurring during the time period under consideration.

MEAN NUMBER OF EQUIPMENT REPLACEMENTS

22. The mean number of equipment replacements expected for a particular item during a specified time interval can be shown to be:

$$M = nk\lambda T \qquad \text{.....Equation \#4}$$

where: n = the number of weapon systems;

k = quantity of a particular item installed in each weapon system;

λ = Lambda, the failure (i.e. replacement) rate; and

T = the time interval (i.e. duration of deployment).

Note: λ and T must be expressed in the same time units, for example, failures per hour and hours of operation.

Also: $\lambda = \frac{1}{MTBF}$ .....Equation #5

where: MTBF = Mean Time Between Failure; in this case, only those failures resulting in repair by replacement are counted.

23. The equations in this report can refer either to a deployment or to sample data. In order to prevent confusion, capital letters, such as M,V,T,P and S are used to refer to deployment values. Small letters, such as m, λ , and k, are used to refer to sample data values.

24. The expected value M, given by equation #4 can be estimated by the historical value of m given by equation #1. That is, if the deployment were repeated a large number of times, there would be on average m failures. However, more than m spares are required to ensure that spares are available in those instances when the number of failures is above average. Therefore, multiplying the planned flying hours by the expected number of failures (i.e. equipment replacements) per flying hour is not an adequate way of determining how many spares are required. It is necessary to take probability distributions into account.

PROBABILITY OF HAVING ENOUGH SPARES

25. In calculating the number of spares required to support a deployment, it is necessary to select a target value for P. When doing so an understanding of the relevant probability theory is helpful.

26. The probability of two independent events occurring is the product of the two individual probabilities. That is, P(A and B) = P(A) P(B) (Ref. G). Thus, for a pack-up kit containing Z items, the probability of having enough pack-up kit spares is the product of each of the individual probabilities P_i of having a spare available for item i when required:

$$P_{Kit} = P_1 \cdot P_2 \cdot P_3 \ldots P_Z \qquad \text{.....Equation \#6}$$

27. Since each P_i is a fraction, the value of P_{Kit} can become quite small. With 20 items in a kit, and P=0.90 for each item, the value of P_{Kit} would be $(0.90)^{20} = 0.12$. So even with a high degree of confidence that there will be no stock-out of any individual item, the confidence against stock-out for the kit as a whole can be low. To achieve a probability of 0.90 that there would be no stock-out of any

item in a kit of 20 line items, each individual item must have a probability of no stock-out high enough so that the product of the probabilities is at least $P_{Kit} = 0.90$.

28. The simplest method of choosing an item target probability is (using this example) to take the 20th root of the kit target. That is,

$$P_i = (0.90)^{1/20} = 0.995$$

With this approach, all items are treated equally.

29. An alternative method would be to choose higher values of P for the most critical items, keeping in mind that the product of the values chosen should equal the target value of P_{Kit}.

30. The remaining question is how to choose a value for P_{Kit}, or in other words how to decide:

"In what percentage of the deployments must there be no stock-out of pack-up kit items?"

The answer to this question involves a trade-off between the operational requirements and the unit's willingness and ability to commit resources. One hundred percent assurance against stock-out would require infinite resources and is therefore, not achievable. On the other hand, a low assurance, such as 50%, would not be acceptable because of the negative impact on operations. A value in-between can be chosen subjectively by considering both the operational need and the resource constraints. Depending upon kit size, a target value such as 0.80 or 0.85 may be reasonable.

31. Figure 1 illustrates the relationship between the operational objective, expressed as 'weapon system operational availability' and resource commitment expressed as 'total investment in spares'. Note that there is some chance that no failures will occur of mission critical equipment and, thus, that no spares will be needed. Therefore, operational availability can be positive with zero investments in spares. Operational availability then increases rapidly with increasing investment in spare parts. In this region, the curve can be almost linear. Finally, diminishing return on investment becomes evident as the curve bends and asymptotically approaches some value less than one. This value is dependent upon factors other than the provision of spare parts, such as repair times and administrative delays. A reasonable target weapon system availability, A_T, is shown. This can be intuitively translated into a target assurance of having enough pack-up kit spares, P_{Kit}, bearing in mind that the pack-up kit does not include all of the components in the weapon system.

32. If the base does not have sufficient stock to provide the desired quantity of spares, the predicted kit availability can be computed using equation #6. Then this value can be used to demonstrate the need for corrective action.

33. If assets must be shared between operations from home base and operations from a remote location, or between simultaneous deployments, equation #6 also can be used to assess alternative distributions of spares.

34. It is important for the pack-up kit to contain as wide a range of items as is practical. This point is illustrated by considering a pack-up kit containing 100 different items that is used to support an aircraft containing 20,000 components.

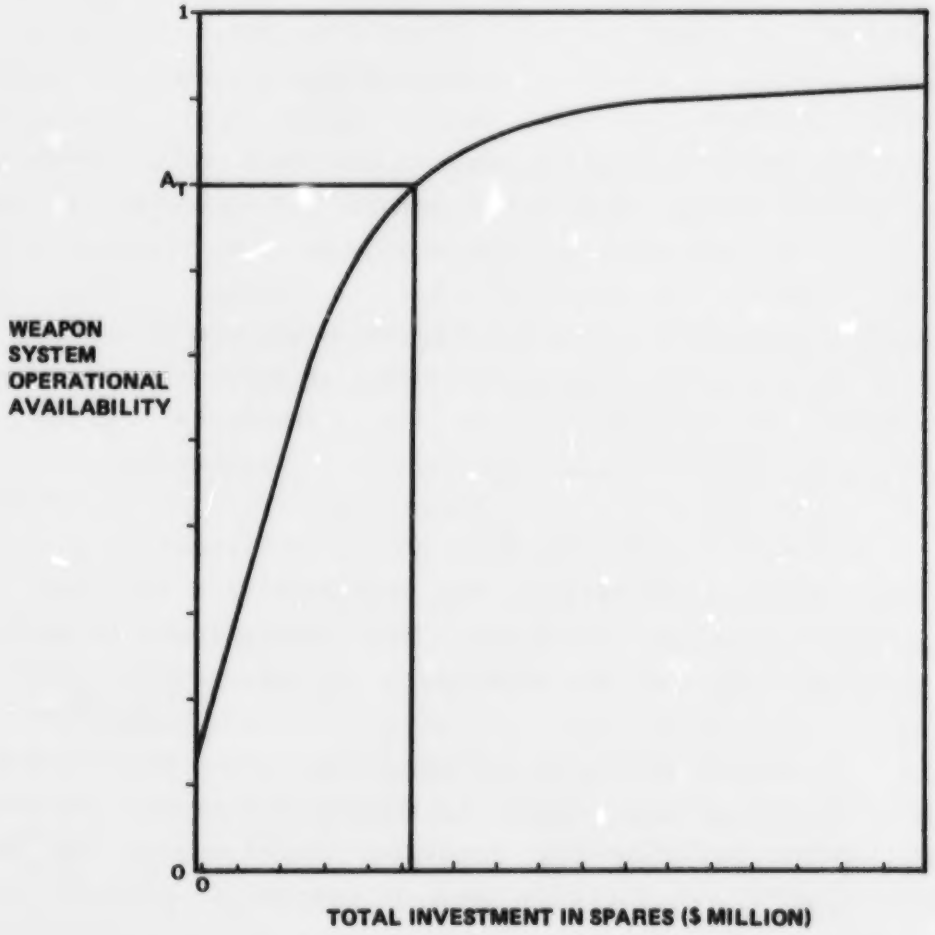


Figure 1: Trade off between Weapon System Operational Availability and Total Investment in Spares. (Adapted from Ref. Q)

There is a good chance that at least one or two of the excluded items will be needed on a particular deployment, even though the failure rate of each of these items is very low.

35. Even the best designed pack-up kit may not be able to meet the requirements of every deployment. Shortages and surpluses are an inevitable consequence of the random nature of equipment failure. An unexpected rash of failures could result in a shortage of some item in the kit. The need to include safety stock will result in a surplus. An example given in Annex A paragraph 31 concerns a pack-up kit with 100 items, each of which has an expected demand rate of only one per deployment. Assuming a Poisson distribution and two of each item in the kit, one would expect a shortage of eight items and a surplus of 110 items. The shortages do not negate the theory, they merely highlight the fact that the planning process involves risk. Over the long run, the risk should be lower if failure data are used than if they are not.

EXAMPLE PROBLEM - POISSON DISTRIBUTION

36. The theory regarding the Poisson distribution is illustrated by the following problem:

Problem 1

37. Consider an aircraft radar receiver-transmitter, which is a line replaceable unit (LRU), that has a mean time between failure of 100 hours. (An LRU is an item that can be replaced during servicing level maintenance to rectify an aircraft unserviceability.) Assume all failures are rectified by equipment replacement. Examination of the failure data reveals a variance to mean ratio of one. Each aircraft has one such receiver-transmitter installed. Twenty aircraft are each scheduled for a ten-hour mission.

38. Ignoring any equipment repair capability, how many spares should be carried in inventory to ensure a 90% chance of having a spare available to rectify each failure? If only three spares are available, what is the probability of having a spare when needed?

39. This problem is solved first by doing detailed calculations and then by using a nomograph. The former is presented for explanatory purposes; the latter method is recommended for use by maintenance personnel.

Solution Using Equations

40. First e^{-M} can be calculated as shown in steps 1,2,3. Then the Poisson calculations can be performed by increasing the number of spares (x) one at a time until a quantity (S) is reached which results in the desired probability (P) being achieved, as shown in step #4.

Step 1: Compute failure rate:

$$\lambda = \frac{1}{MTBF} = \frac{1}{100} = 0.01$$

Step 2: Compute mean number of failures:

$$M = nk\lambda T = (20)(1)(.01)(10) = 2$$

Step 3: Compute the probability of no failures:

$$e^{-M} = e^{-2} = 0.135$$

Step 4: Compute spares required, probability of no stock-out:

TERM	INCREASE IN P	P	S	TARGET
$\frac{0.135(2)^0}{0!}$	0.135	0.135	0	
$\frac{0.135(2)^1}{1!}$	0.270	0.405	1	
$\frac{0.135(2)^2}{2!}$	0.270	0.675	2	
$\frac{0.135(2)^3}{3!}$	0.180	0.855	3	
		.....	.....	.. 0.90
$\frac{0.135(2)^4}{4!}$	0.090	0.945	4	

41. For those not familiar with factorial notation, $3! = 3 \times 2 \times 1$; $1! = 1$; also, $0! = 1$. For those not familiar with exponential notation $2^3 = 2 \times 2 \times 2$; $2^1 = 2$; $2^0 = 1$.

42. It can be seen from the above table that four spares are required to meet the 0.90 target. If only three spares are available, then the assurance level is P = 0.855.

43. The concepts introduced in Figure 1 are reinforced by the above table, which was used to draw the curve shown in Figure 2. Note that each additional spare contributes an increase in P, but that the increase gets progressively smaller. That is, each additional spare is less valuable than its predecessor. This fact is taken into account when there are cost, weight or volume limitations.

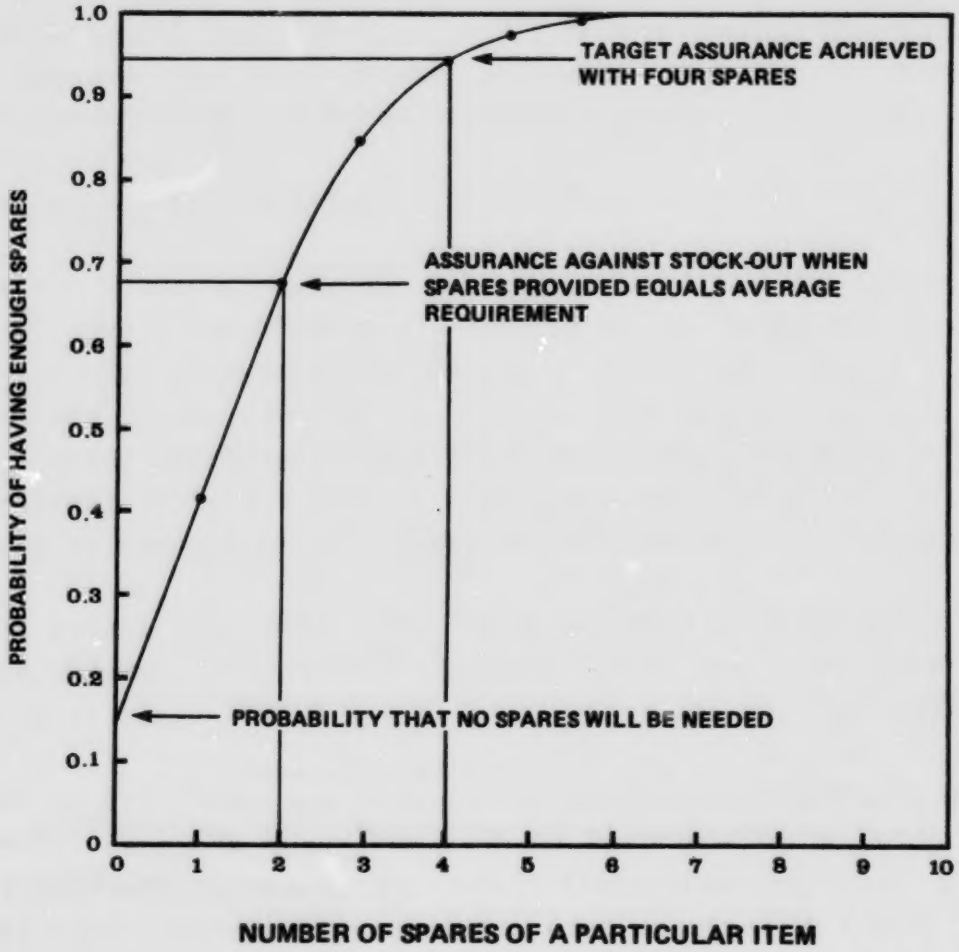


Figure 2: A Graph Showing the Diminishing Benefit of Additional Spares

Solution Using Nomograph

44. Performing the Poisson calculations by hand can be tedious. At Canadian Forces bases, use of a nomograph is more suitable.

Step 1: Compute failure rate:

As before, = 0.01

Step 2: Compute expected mean number of failures:

As before, M = 2

Step 3: a) Compute spares required (see Figure 3):

Join M=2 to P=0.90 and read S.
Round S up to next larger integer.

b) Compute probability of no stock-out (see Figure 3):

Join M=2 to S=3 and read P.

45. The answers are S=4, and if S=3, P=0.855. Such answers can be obtained very quickly using a transparent straight edge and nomograph.

A COMPOUND POISSON DISTRIBUTION

46. If the data on equipment replacements exhibit a variance to mean ratio (R) greater than one, then the simple Poisson distribution will not give a good estimate of the spares required to ensure against stock-out. If it were used, the quantity of spares indicated would be too low. A Compound Poisson distribution will produce better results. This can be

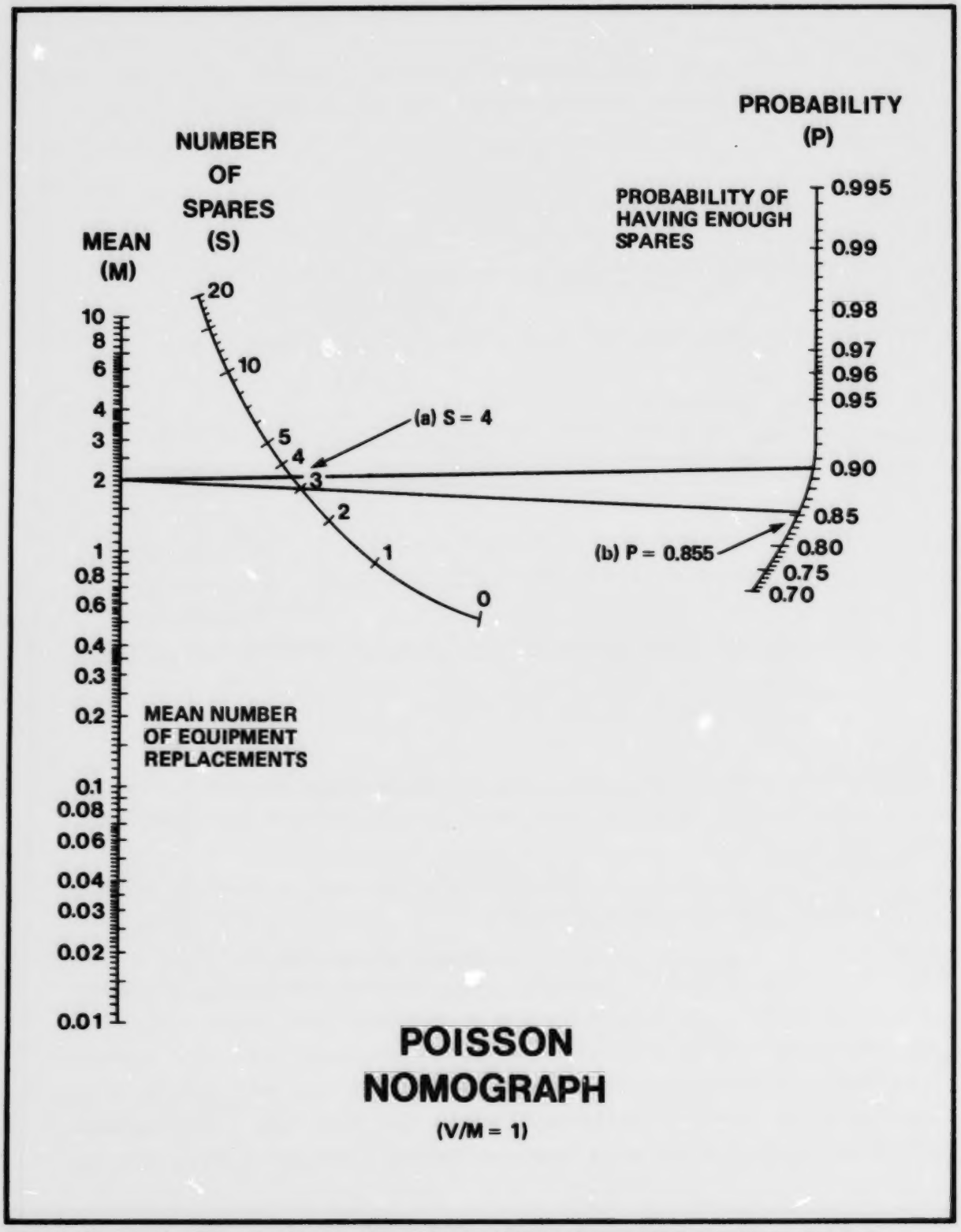


Figure 3: Nomograph to Determine Quantity of Spares Required Based on Poisson Distribution (Source: Ref. B, as reprinted in Ref. C, p. 56, but corrected and adapted.)

seen in Figure 4 in which a line is drawn joining a specific value for M and P and observing the values of S for different values of R. For example, if M=0.4 then S = 3 spares would be required to meet the P=0.995 assurance target using the Poisson model (i.e. R=1). If the equipment replacements correspond to a Compound Poisson distribution with R=2, then S = 4 spares are required to meet the same target. Similarly, if R=3, S=6 and if R=5, S=8.

47. Several slightly different Compound Poisson distributions are described in literature on inventory theory. The Negative Binomial form was selected by the authors; its distribution function is given in Annex A. Since this function is difficult to use in computations, an alternative step-by-step approach has been developed which is described below.

48. First compute the variance to mean ratio:

$$R = V/M \qquad \text{.....Equation \#7}$$

Then, for R greater than 1:

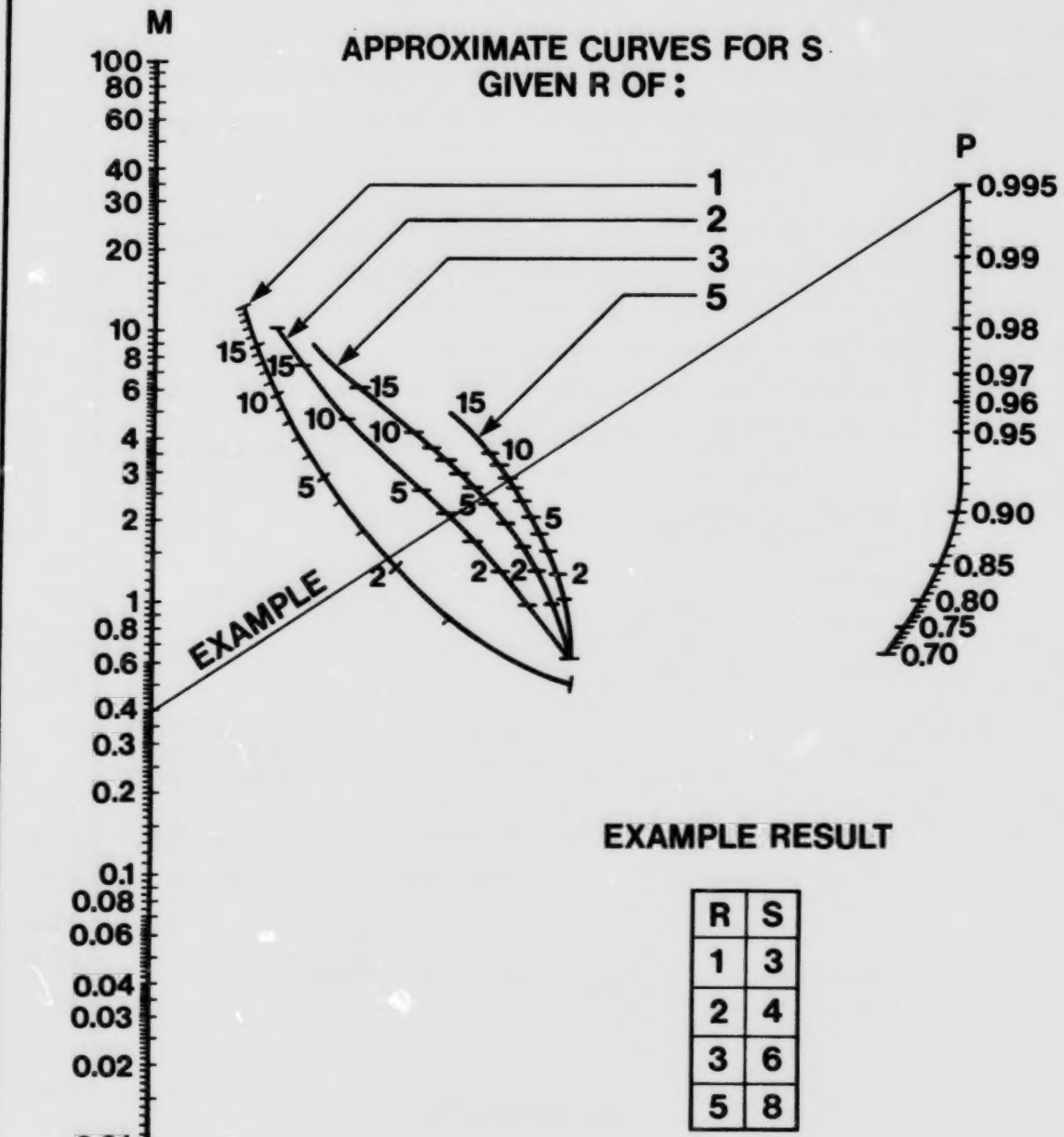
$$P(0)=\frac{1}{R^{y}} \qquad \text{.....Equation \#8}$$

where P(0) is the probability that no spares will be needed. To calculate P(1), P(2), P(3), ... one must first compute

$$y=\frac{M}{R-1} \qquad \text{.....Equation \#9}$$

Then,

$$P(x+1)=\frac{P(x)\cdot(x+y)\cdot(R-1)}{(x+1)\cdot(R)} \qquad \text{.....Equation \#10}$$



EXAMPLE RESULT

R	S
1	3
2	4
3	6
5	8

Figure 4: The Effect of the Variance to Mean Ratio (R) on the number of Spares Required

Note that the calculation for (x+1) is based on the calculation for x. Thus, equation #10 is used repeatedly for ever-increasing values of x until the desired level of P is achieved. In other words:

$$P_i = \sum_{x=0}^{S} P(x) \qquad \text{.....Equation \#11}$$

where S is the number of spares in the kit.

EXAMPLE PROBLEM - COMPOUND POISSON

49. Computations using the Compound Poisson distribution are very tedious to do by hand. However, an example, Problem 2 below, is provided to ensure that the reader has an understanding of the equations. The solution is repeated using a nomograph that greatly simplifies the computation. The nomograph method is recommended for use by aircraft maintenance personnel.

Problem 2

50. Consider an aircraft system consisting of one line replaceable unit (LRU) per aircraft. The failure data given below were obtained by dividing the past two years of operations into intervals of 300 flying hours and then determining the number of failures per interval.

FAILURES DURING EACH INTERVAL
OF 300 FLYING HOURS

2	1	3	9	1	0	3	0	5	1
0	0	5	2	0	6	1	3	1	7
4	2	0	3	8	0	0	5	0	0
1	0	1	0	0	1	8	1	4	5
6	0	0	2	7	0	0	2	0	4

51. Four aircraft are being sent on a deployment. Each aircraft is scheduled to fly 90 hours, including transit time. Aircraft repair will be by LRU replacement. Removed equipment will not be repaired during the deployment.

52. How many spares should be taken on the deployment to ensure a 0.95 probability of having enough spares? If only four spares are available, what is the probability of having a spare when needed?

Solution Using Equations

Step 1: Compute m,v,R:

53. Computation of the mean and variance of the sample data is shown in the table below. However, some of the data are omitted to save space. Many hand-held calculators have built-in programs which do this calculation.

	FAILURES	MEAN	DEVIATION	SQUARED DEVIATION
	2	2.28	-0.28	0.08
	0	2.28	-2.28	5.20
	4	2.28	1.72	2.96
	1	2.28	-1.28	1.64
	6	2.28	3.72	13.84
	.	.	.	.
	.	.	.	.
	.	.	.	.
	1	2.28	-1.28	1.64
	7	2.28	4.72	22.28
	0	2.28	-2.28	5.20
	5	2.28	2.72	7.40
	4	2.28	1.72	2.95
TOTALS:	114			332.08
	÷50			÷49
MEAN =	2.280		VARIANCE =	6.777

VARIANCE/MEAN = 6.777/2.280= 2.972

Since the variance to mean ratio is significantly larger than one, a Compound Poisson distribution is applicable.

Step 2: Compute mean number of replacements for the deployment:

54. The sample data were obtained using intervals of 300 flying hours, but the planned deployment duration is 360 hours (4 x 90). Therefore, on the average one would expect:

M = 2.280 x (360/300) = 2.736 failures.

Note that a similar adjustment is applicable whenever the interval used in sampling the data is different from the deployment duration. If the difference is large, new sample data should be obtained using an interval length equal to the deployment duration.

Step 3: Compute Y and (R-1)/R:

55. The following equations apply.

$$Y = \frac{M}{R-1} = \frac{2.736}{(2.972-1)} = 1.387; \quad \frac{R-1}{R} = \frac{(2.9772-1)}{2.972} = 0.664$$

Step 4: Compute spares required or probability of no stock-out:

56. Computations are presented in the table below.

TERM	INCREASE IN P	P	S	TARGET
$\frac{1}{(2.972)^{1.387}}$	0.220	0.220		
$\frac{(0.220)(0 + 1.387)(0.664)}{(0 + 1)}$	0.203	0.423	1	
$\frac{(0.203)(1 + 1.387)(0.664)}{(1 + 1)}$	0.161	0.584	2	
$\frac{(0.161)(2 + 1.387)(0.664)}{(2 + 1)}$	0.121	0.705	3	
$\frac{(0.121)(3 + 1.387)(0.664)}{(3 + 1)}$	0.088	0.793	4	
$\frac{(0.088)(4 + 1.387)(0.664)}{(4 + 1)}$	0.063	0.855	5	
$\frac{(0.063)(5 + 1.387)(0.664)}{(5 + 1)}$	0.045	0.900	6	
$\frac{(0.045)(6 + 1.387)(0.664)}{(6 + 1)}$	0.032	0.932	7	
		.....		0.95
$\frac{(0.032)(7 + 1.387)(0.664)}{(7 + 1)}$	0.022	0.954	8	

Thus eight spares are required to meet the 0.95 assurance target. If only four spares are available, the probability of having enough spares is only 0.79.

Solution Using Nomograph

57. Figure 5 provides a Compound Poisson nomograph that greatly simplifies the computations, as shown below:

Step 1: Compute m,v,R:

As before, m=2.280,v=6.777,R=2.972.

Step 2: Compute expected value for the deployment:

As before, M=2.736.

Step 3: Choose the appropriate nomograph:

Since R=2.972, use the nomograph developed for R=3.

Step 4: a) Compute spares required (see Figure 5):

Join M=2.74 to P=0.95 and read S off the curve.

b) Compute probability of no stock-out (see Figure 5):

Join M=2.74 to S=4 and read P.

58. Note that the same answer is obtained as was previously calculated. However, the more tedious calculations were avoided.

59. In Figures 6 through 9 inclusive, nomographs are presented for variance to mean ratios of R=1,2,3 and 5 respectively. For the reader's convenience, the same figures are repeated in the User's Instructions - Annex C.

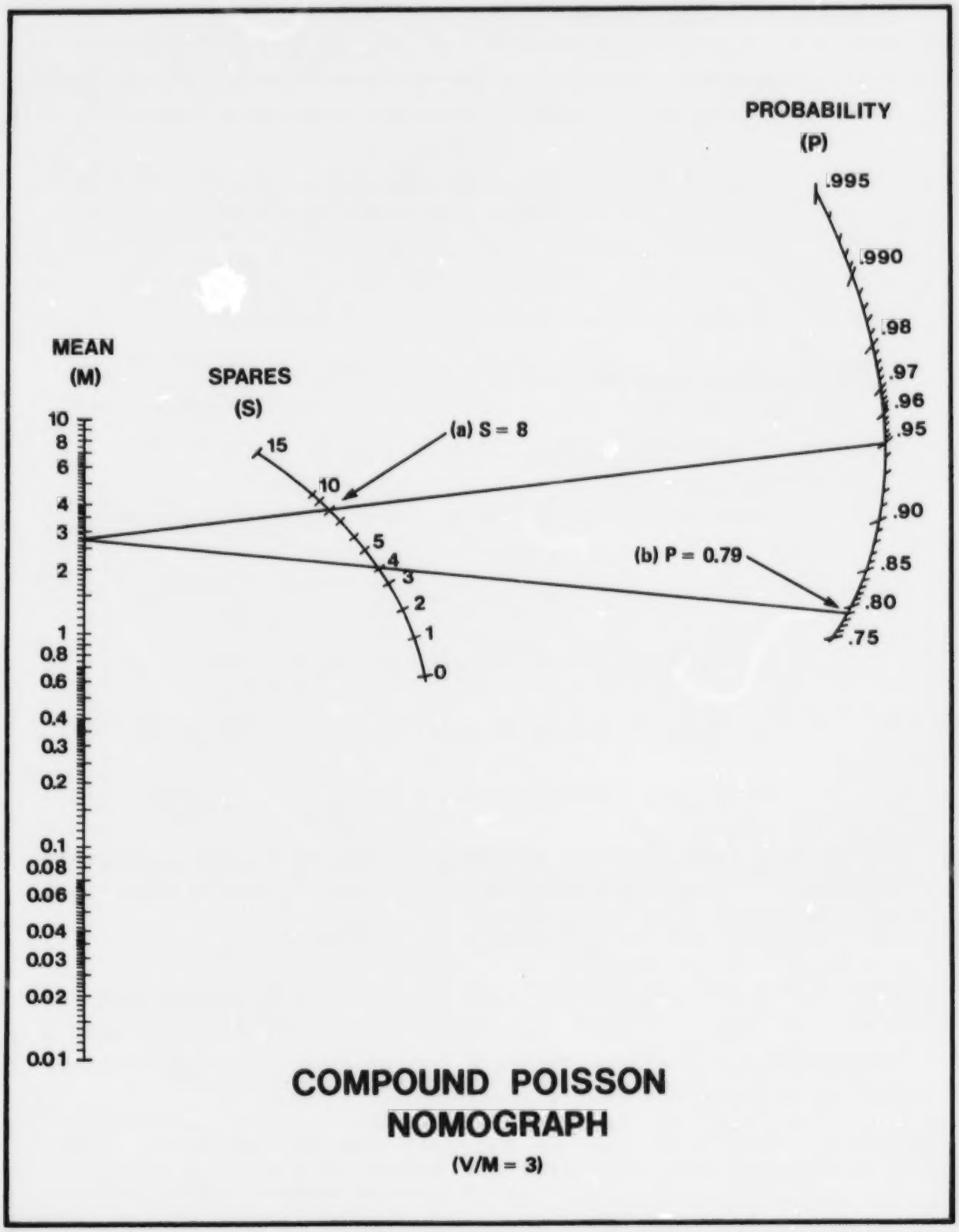


Figure 5: Nomograph to Determine Quantity of Spares Required Based on Componed Poisson Distribution for V/M = 3

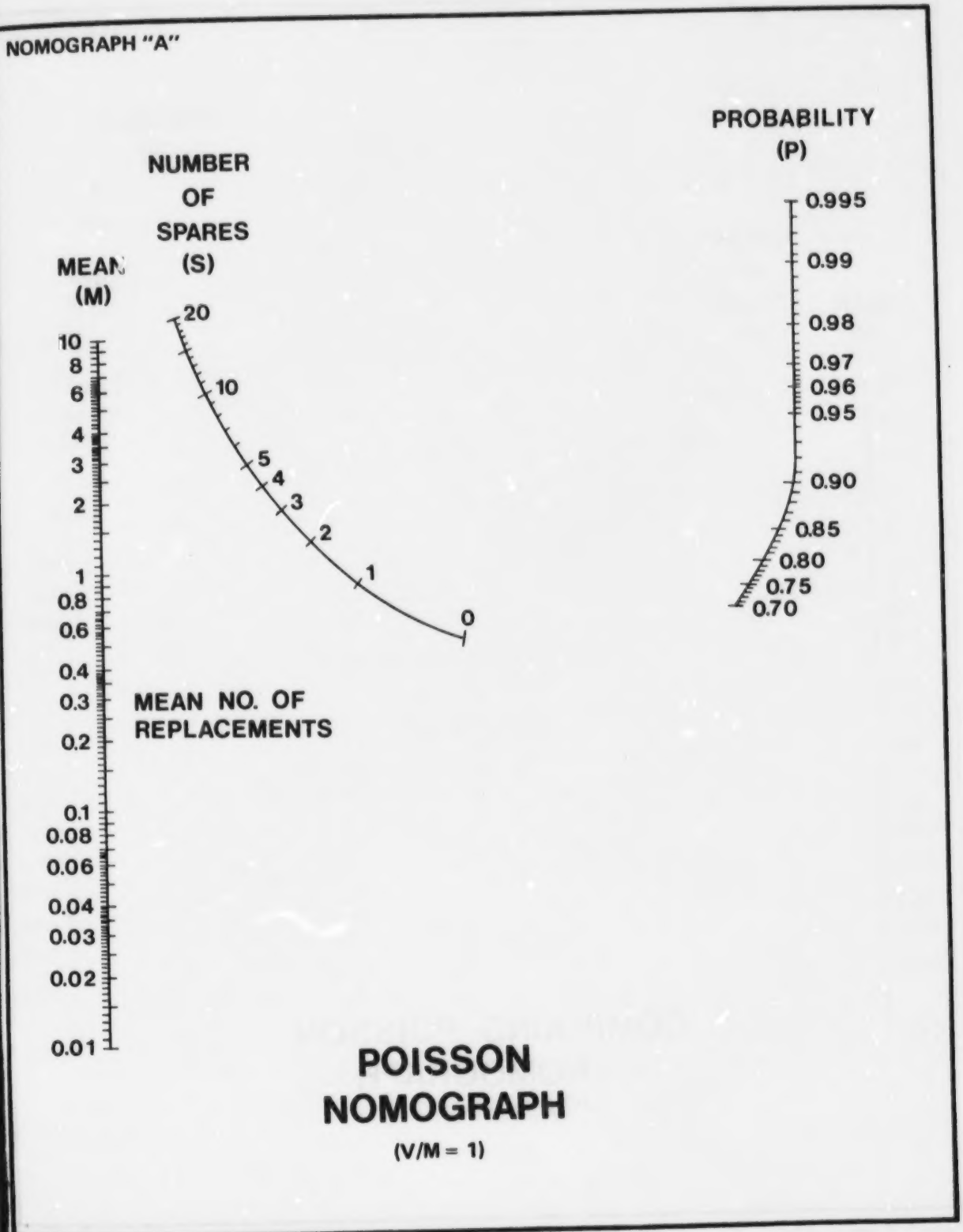


Figure 6: Poisson Nomograph Calibrated at Variance/Mean Ratio = 1

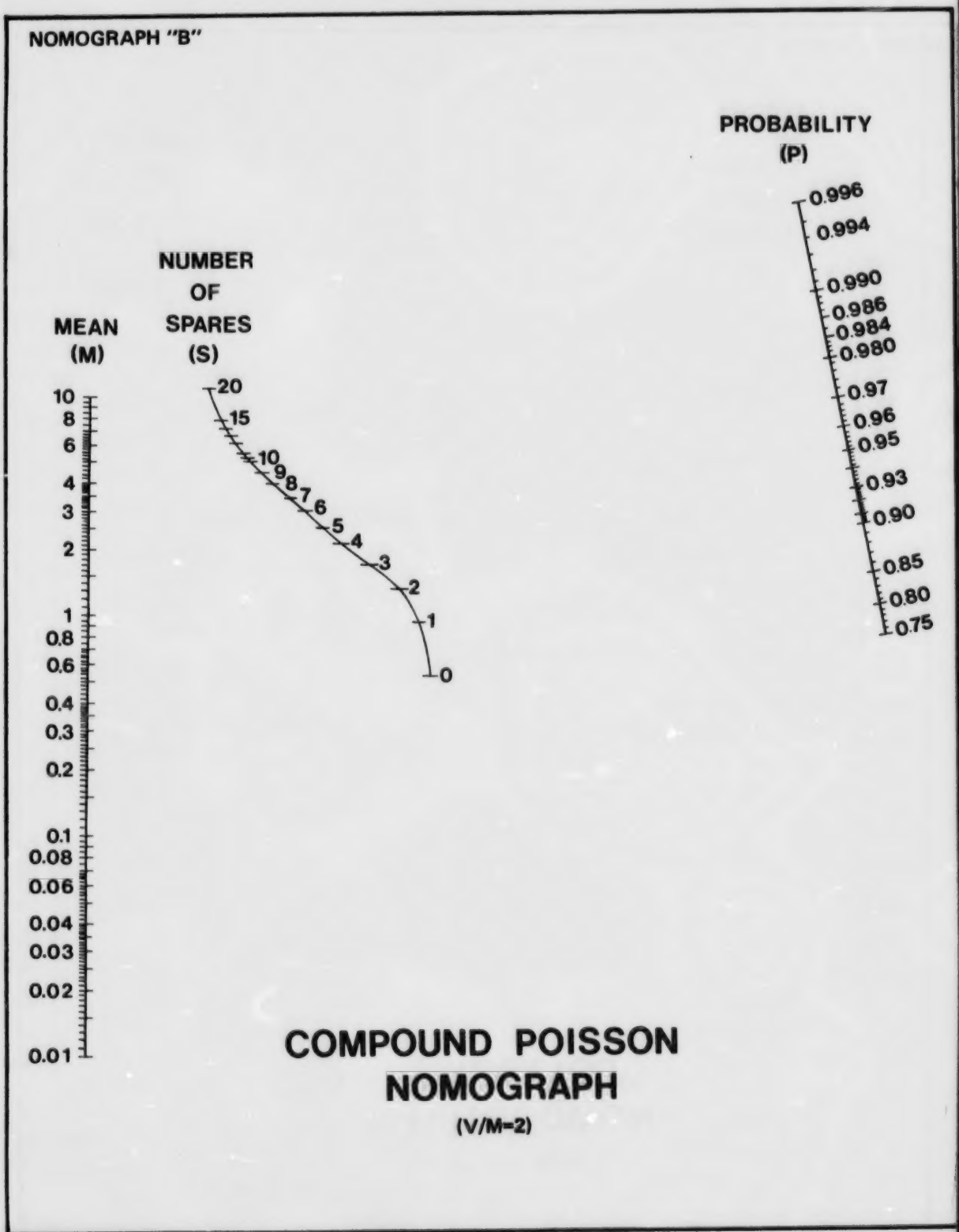


Figure 7: Compound Poisson Nomograph Calibrated at Variance/Mean Ratio = 2

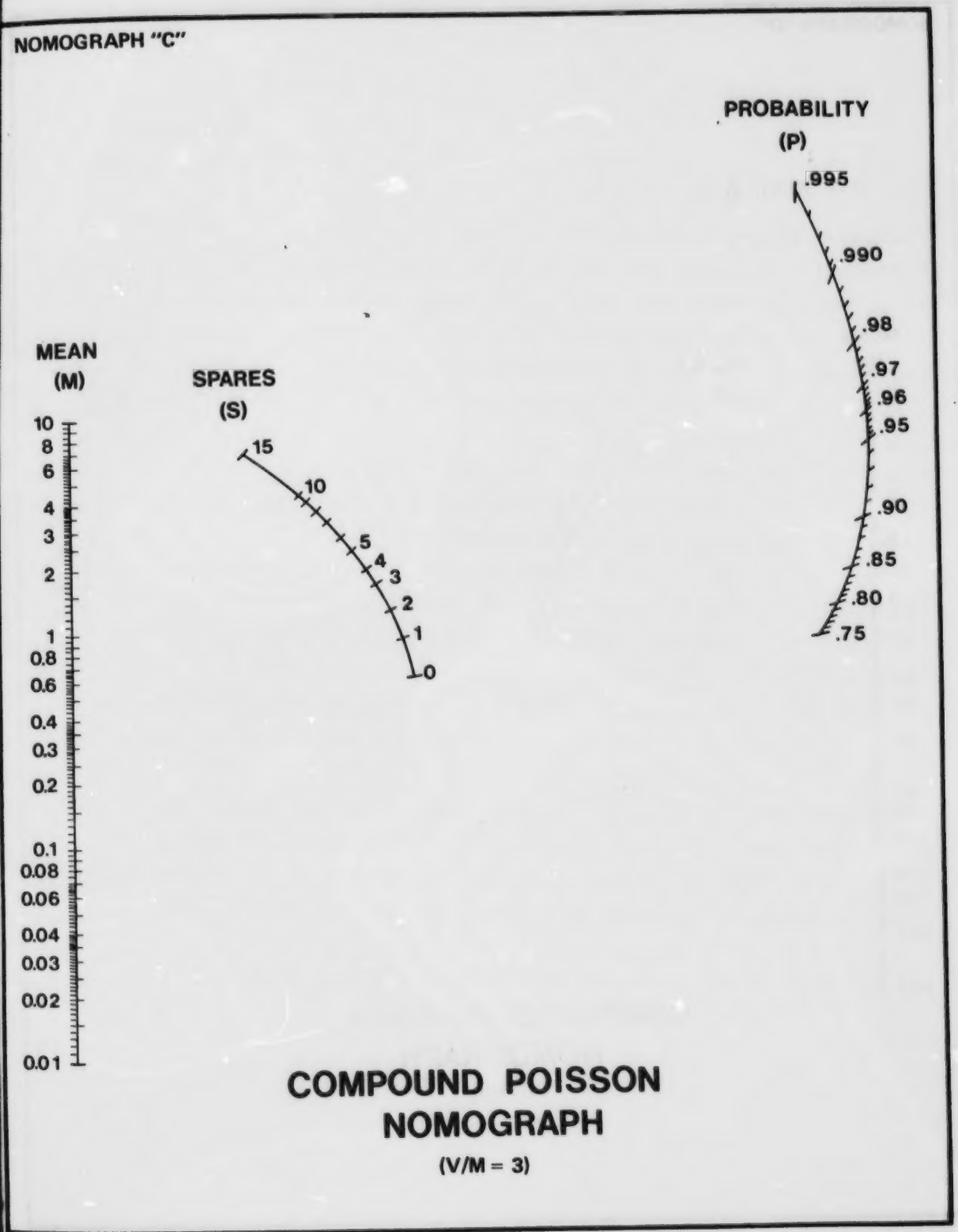


Figure 8: Compound Poisson Nomograph Calibrated at Variance/Mean Ratio = 3

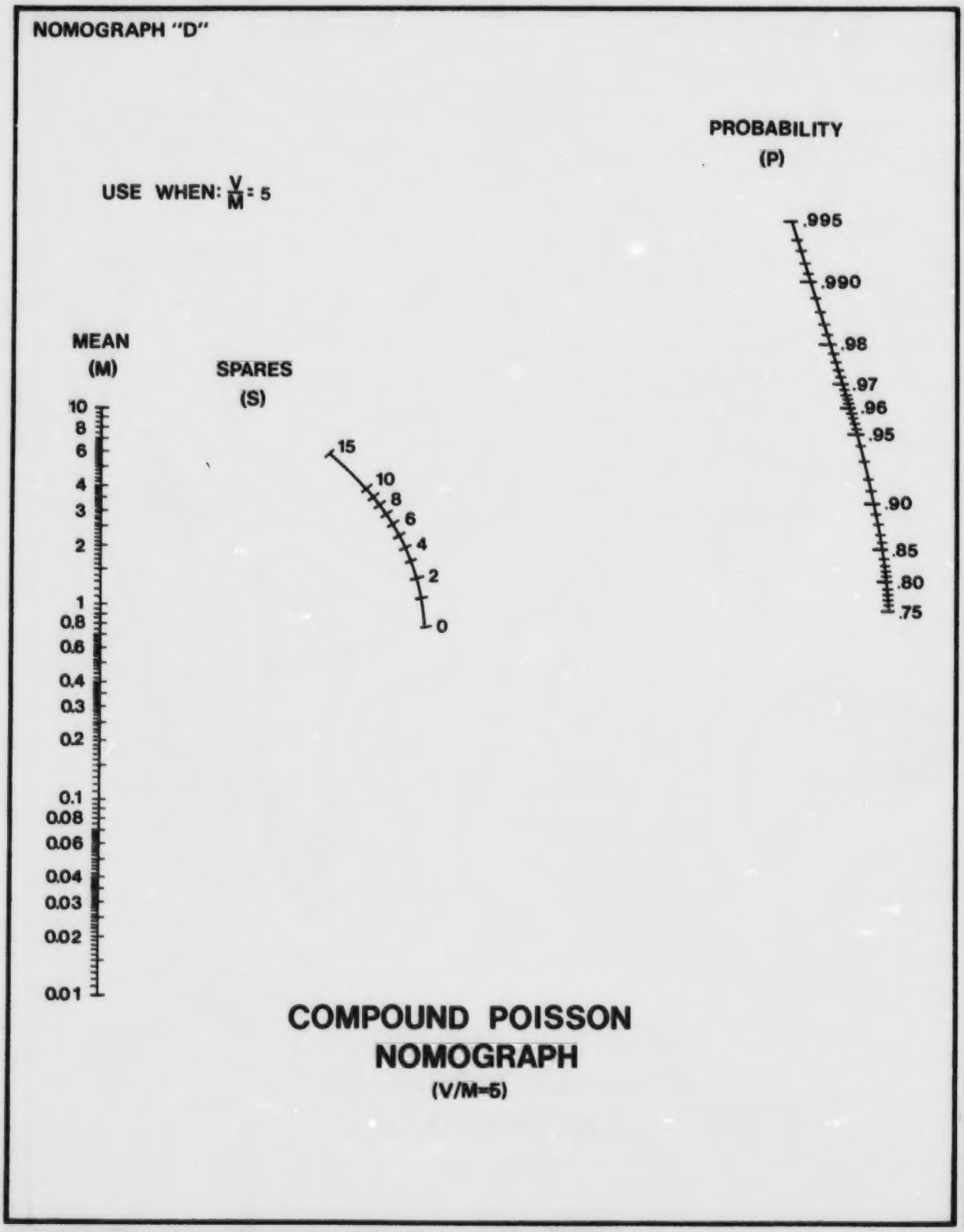


Figure 9: Compound Poisson Nomograph Calibrated at Variance/Mean Ratio = 5

DATA EXTRACTION TECHNIQUE

60. The sampling technique is based on viewing aircraft operations as a continuous series of deployments. For this report, equipment replacement data from the past six years of CF116 operations were divided into successive intervals of 450 flying hours. An advantage of this method is that fluctuations in fleet size or flying rate do not affect the data.

61. Figure 10 illustrates the computer output. In the scattergram, one point is plotted for each of the 241 intervals. Since the plot was created by a line printer with low resolution, adjacent points often seem to be one above the other. Also, if two adjacent points have the same value, a "2" is printed, and similarly for "3". The histogram is straight forward. The meaning of the various terms in the statistical summary is discussed later. However, note that values for the mean and variance are provided.

62. If there is more than one aircraft configuration in a given fleet, for example, CF116 single and dual seat aircraft, then a computer run is performed for each configuration. This takes into account differences in quantity of items installed.

63. A complete explanation of how the data were obtained is given in Annex B.

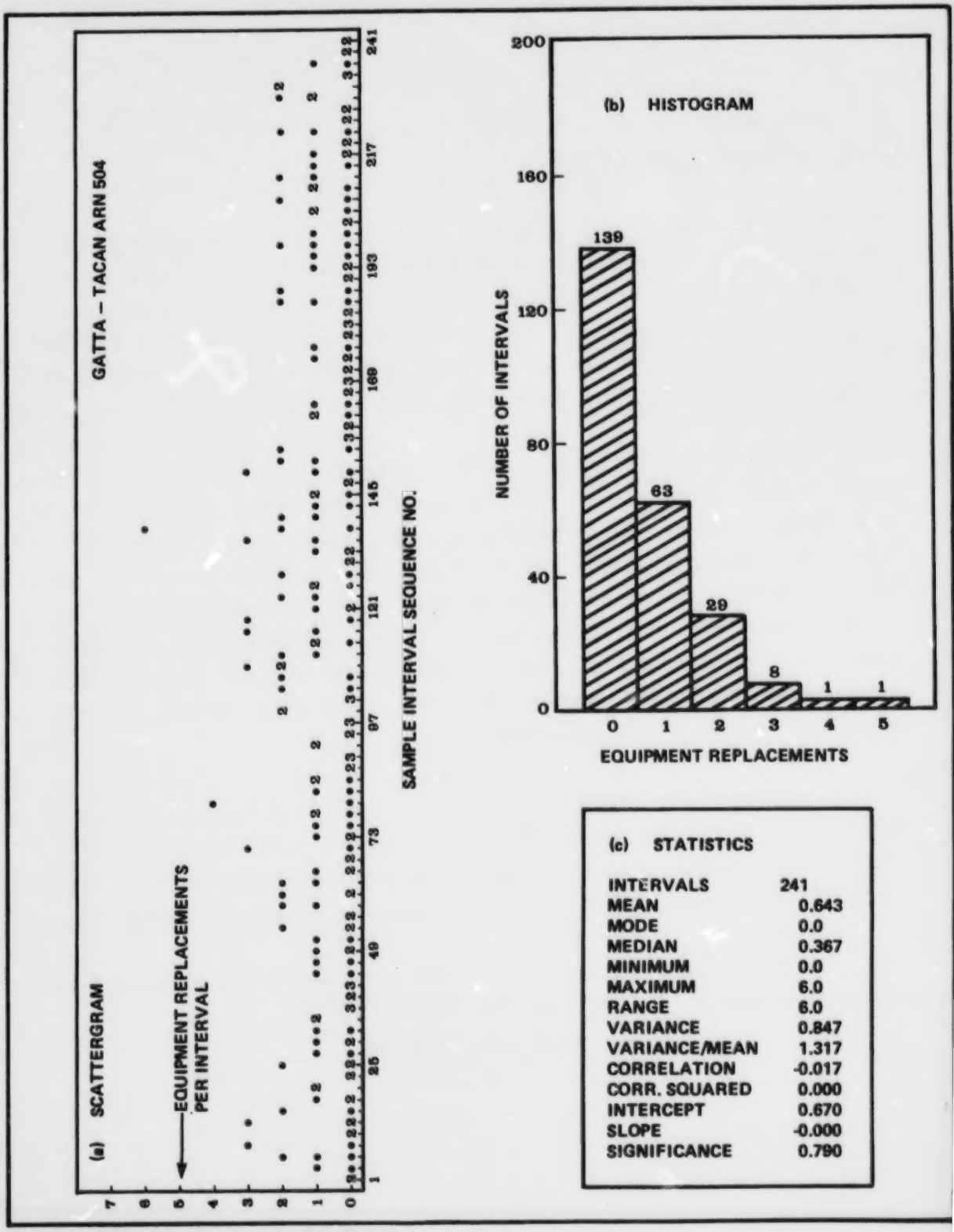


Figure 10: Output from Statistical Analysis Program

A PRACTICAL EXAMPLE - CF116 PACK-UP KIT

64. Consider the following 'real life' problem:

Problem 3

65. 434 Squadron will deploy eight CF116 single seat aircraft on a one-month mission. They will fly a total of 450 hours, including transit time. Spare parts will be taken in a pack-up kit to support the deployment. There will be no off-aircraft equipment repair capability, so enough spare parts must be taken to meet requirements for repair by replacement. The items to be included in the pack-up kit are listed in Table I. They are identified by work unit code (WUC) and nomenclature. Table II provides the equipment replacement data (M,V,R) for these items.

66. Determine the quantity of spare parts required to ensure that the probability of having enough of each spare is 0.99.

Solution

67. A solution was obtained following the procedures given in Annex C - User's Instructions. Since nomographs are provided only for variance to mean ratios of one, two, three and five, it was necessary to select the appropriate nomograph to use for each computation. As explained in Annex C, the rule applied was to choose the nomograph with closest R value. For example, for the ven leaf (outer) R=2.9, so the nomograph for R=3 was used.

68. The solution is given in Table III. An example computation is shown in Figure 11 for the canopy seal.

TABLE I

CF116 SINGLE SEAT AIRCRAFT

PACKUP KIT ITEMS

NO.	WUC	NOMENCLATURE	INSTALLED QUANTITY
1*	ABBF	CANOPY BALANCE CABLE	2
2*	AHAA	BRAKE ASSEMBLY	2
3	AHBE	STEERING ACTUATOR	1
4	AHGL	MAIN UNDERCARRIAGE HYDRAULIC LINE	1
5	BEKM	VEN LEAF (OUTER)	12
6	BEJE	FLAME HOLDER	2
7*	BEM	ACCESSORY DRIVE SYSTEM	2
8	BESA	VEN POWER UNIT	2
9	BEYK	IGNITOR PLUG (AFTER BURNER)	2
10	DAEB	CANOPY SEAL	1
11**	EBCB	NOZZLE POSITION INDICATOR	2
12	EDA	STABILITY AUGMENTATION SYS.COMPUTER	1
13	EJBC	GHARS ELECTRONIC UNIT	1
14	EJBE	GHARS CONTROL	1
15	EJBV	MAIN ATTITUDE INDICATOR	1
16	FEETB	UHF RECEIVER-TRANSMITTER	1
17	FGEXB	IFF RECEIVER-TRANSMITTER	1
18	FHETA	RECEIVER-TRANSMITTER	1
19**	GATTA	TACAN ARN 504	1
20**	GAURA	ADF RECEIVER	1

*Upper and lower or left and right installations are under the same Work Unit Code.

**Data were extracted only for CF116 single seat configuration.

TABLE II

CF116 SINGLE SEAT AIRCRAFT

EQUIPMENT REPLACEMENT DATA

No.	WUC	NOMENCLATURE	M	V	R
1	ABBF	CANOPY BALANCE CABLE	1.531	3.275	2.1
2	AHAA	BRAKE ASSEMBLY	1.108	2.022	1.8
3	AHBE	STEERING ACTUATOR	0.357	0.439	1.2
4	AHGL	MAIN UNDERCARRIAGE HYDRAULIC LINE	0.232	0.271	1.2
5	BEKM	VEN LEAF(OUTER)	2.158	6.175	2.9
6	BEJE	FLAME HOLDER	2.017	3.025	1.5
7	BEM	ACCESSORY DRIVE SYSTEM	2.548	5.474	2.2
8	BESA	VEN POWER UNIT	1.722	3.043	1.8
9	BEYK	IGNITOR PLUG (AFTER BURNER)	2.170	4.142	1.9
10	DAEB	CANOPY SEAL	0.129	0.129	1.0
11	EBCB	NOZZLE POSITION INDICATOR	0.381	0.340	0.9
12	EDA	STABILITY AUGMENTATION SYS. COMPUTER	0.502	0.626	1.3
13	EJBC	GHARS ELECTRONIC UNIT	2.054	3.426	1.7
14	EJBE	GHARS CONTROL	1.817	2.225	1.2
15	EJBV	MAIN ATTITUDE INDICATOR	0.701	0.844	1.2
16	FEETB	UHF RECEIVER-TRANSMITTER	4.191	6.730	1.6
17	FGEXB	IFF RECEIVER-TRANSMITTER	1.971	3.313	1.7
18	FHETA	RECEIVER-TRANSMITTER	0.307	0.330	1.1
19	GATTA	TACAN ARN 504	1.194	1.515	1.3
20	GAURA	ADF RECEIVER	0.284	0.337	1.2

TABLE III

CF116 SPARES REQUIRED TO SUPPORT

450 FLYING HOURS DEPLOYMENT

NO.	WUC	NOMENCLATURE	M	R	P	S
1	ABBF	CANOPY BALANCE CABLE	1.531	2	0.99	8*
2	AHAA	BRAKE ASSEMBLY	1.108	2	0.99	6
3	AHBE	STEERING ACTUATOR	0.357	1	0.99	2
4	AHGL	MAIN UNDERCARRIAGE HYD. LINE	0.232	1	0.99	2
5	BEKM	VEN LEAF (OUTER)	2.158	3	0.99	11
6	BEJE	FLAME HOLDER	2.017	2	0.99	9
7	BEM	ACCESSORY DRIVE SYSTEM	2.548	2	0.99	10
8	BESA	VEN POWER UNIT	1.722	2	0.99	8
9	BEYK	IGNITOR PLUG (AFTER BURNER)	2.170	2	0.99	9
10	DAEB	CANOPY SEAL	0.129	1	0.99	1
11	EBCB	NOZZLE POSITION INDICATOR	0.381	1	0.99	2
12	EDA	SAS COMPUTER	0.502	1	0.99	3*
13	EJBC	GHARS ELECTRONIC UNIT	2.054	2	0.99	9
14	EJBE	GHARS CONTROL	1.817	1	0.99	6
15	EJBV	MAIN ATTITUDE INDICATOR	0.701	1	0.99	3
16	FEETB	UHF REC-TRANS	4.191	2	0.99	13
17	FGEXB	IFF REC-TRANS	1.971	2	0.99	9
18	FHETA	RECEIVER TRANSMITTER	0.307	1	0.99	2
19	GATTA	TACAN ARN 504	1.194	1	0.99	4*
20	GAURA	ADF RECEIVER	0.284	1	0.99	2*
P VALUE FOR KIT (Target/Predicted)					0.82	0.89

* For these items, the number of spares required will be reduced if faulty components can be replaced during the deployment.

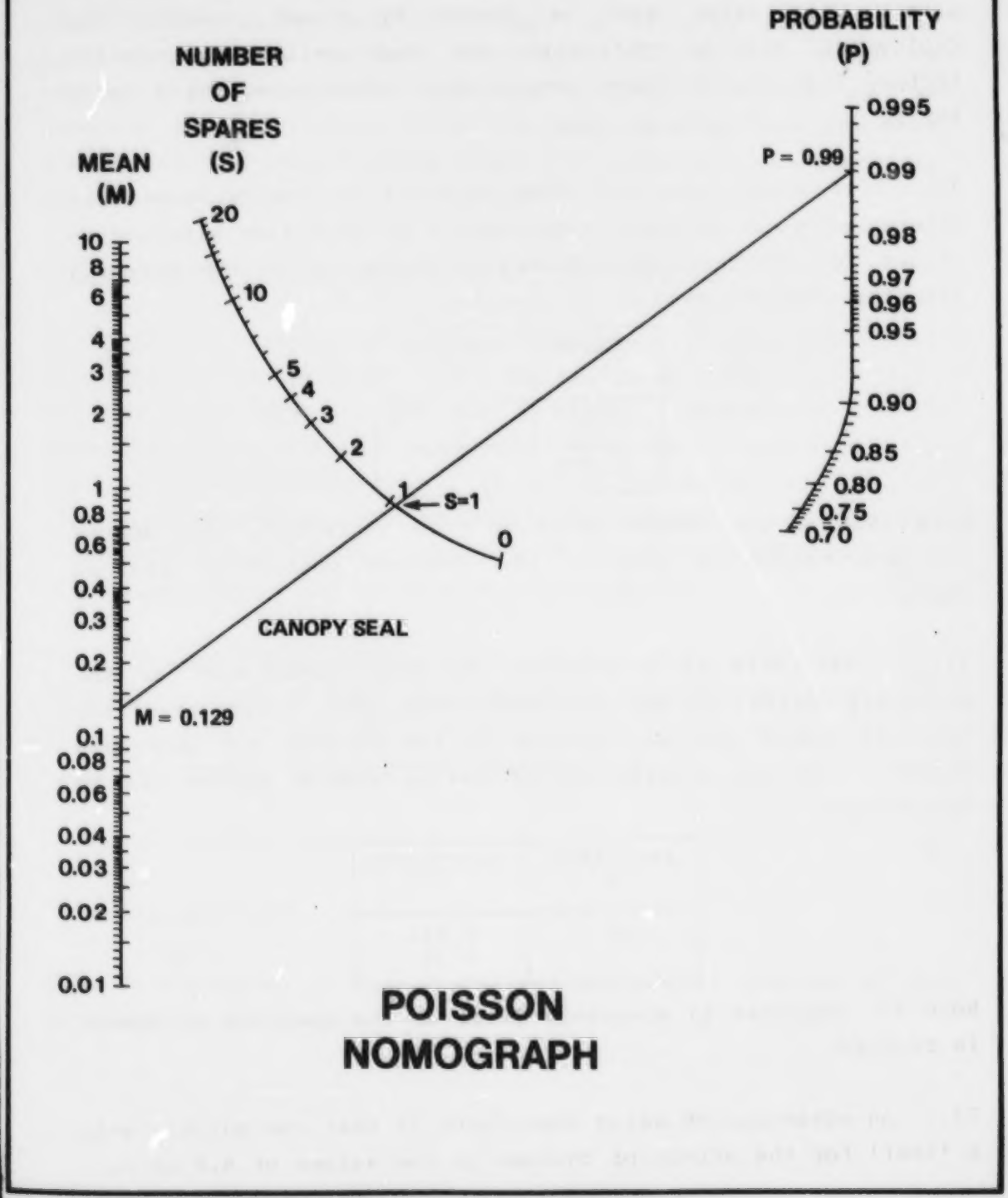


Figure 11: Poisson Nomograph used to Calculate 434 Sqn. Pack-up Kit Spares Quantities

CHOOSING ALTERNATIVE TARGET PROBABILITIES

69. In Problem 3, the item target assurance level (P) was given. This value must be chosen by those planning the deployment. Mission criticality and item availability are two factors that may be taken into account when selecting a value for P.

70. If some items are more critical to the mission than others, this can be taken into account by selecting alternative values for P. This is illustrated below using the steering actuator (M=0.357,R=1) as an example:

TARGET P	REQUIRED S
0.90	1
0.99	2
0.995	3

Note that as the target value of P is increased, more spares (S) are needed to provide the desired confidence against stock-out.

71. If there is a shortage of some item, such as the accessory drive system (M=2.548, R=2) and fewer than the required number can be included in the pack-up kit (say only three), then the probability of having enough spares can be determined:

AVAILABLE S	CALCULATED P
10	0.994
3	0.75

Note the decrease in assurance level as the quantity of spares is reduced.

72. An advantage of using nomographs is that one quickly gets a 'feel' for the effect of changes in the values of M,P or S.

PART III - CONSIDERATIONS WITH ACTUAL DATA

73. When using this technique, it is very important to examine the data to make sure that the Poisson model is appropriate and that the mean and variance generated by the computer are the correct values to use. How to check the data and obtain correct values for the mean and variance is discussed in this part of the report.

VISUAL INSPECTION OF HISTOGRAMS AND SCATTERGRAMS

74. Visual inspection of the data is very important. The data should not be used until it has been carefully vetted. This will be done by DAS Eng 5 staff. Histograms can be inspected to obtain adequate assurance regarding the appropriateness of the model. If the histogram appears to have the same shape as the theoretical distribution, that is, unimodal, skewed to the right, with smooth transitions, then the Poisson model may be used with confidence.

75. Visual inspection of each scattergram is also essential. Three problems have been encountered in data used to date:

a. trends in equipment reliability;

b. cyclic data; and

c. outliers.

Each is discussed in turn below and methods of overcoming the problems are suggested.

TRENDS IN EQUIPMENT RELIABILITY

76. If there is a trend in the number of equipment replacements, then use of the Poisson model with the mean of all of the data will yield an incorrect answer. If the equipment reliability has improved, S will be too large; if the equipment has become less reliable, S will be too small.

77. The CF116 pack-up data were checked to determine if there were any such trends. The data were plotted and the correlation between equipment replacements and time (i.e. sample interval sequence number) was measured. An example is shown in Figure 12. Over six years, the frequency of equipment replacement for this item appears to have increased from 1.5 to 2.9 per 450 flying hours.

78. However, before asserting that a trend really exists, it is necessary to consider two statistical measures used in trend analysis, namely correlation coefficient and significance level.

79. In this application, the correlation coefficient expresses the degree of relationship between the frequency of equipment replacement and the time of observation. A positive correlation coefficient indicates that the frequency of equipment replacement is increasing with time, and a negative coefficient means the opposite. A correlation coefficient of zero indicates no relationship; a coefficient close to plus or minus one indicates a strong relationship. The correlation coefficient squared indicates the percent of the variation in the data that can be attributed to the predictor (time).

80. In Figure 12, the correlation coefficient is 0.20. Since it is positive, it is possible that the frequency of equipment replacement is increasing over time. However, since it is close to zero, this relationship is weak. In fact only 4% (i.e. 0.2^2) of the variance can be attributed to time. The remaining 96% of the variance has some other explanation, including merely random fluctuation.

81. The significance level is the probability that the 'true' correlation coefficient is zero, and that the calculated non-zero value is merely the result of chance outcome. A significance level of less than 0.05 is often accepted as adequate to assert the existence of a trend. With the CF116 data, much smaller significance levels were common. This is because the sample size was sufficiently large to demonstrate even very weak trends. For the data in Figure 12, the significance level is 0.002. Therefore, the correlation coefficient is almost certainly non-zero. In other words, a correlation almost certainly exists but the relationship is weak.

82. One pragmatic approach to this problem would be to ignore this weak trend, allowing it to be taken into consideration by the variance used in the Poisson computations. This approach is applicable to the CF116 pack-up kit, as illustrated by Table IV. Note that the correlation coefficient in all cases is less than 0.30.

83. When the correlation coefficient is large, say above 0.7, the existence of a trend must be considered when computing the required number of spares. It may even be worthwhile for moderate values, say above 0.4. In these cases the expected value for the required number of spares is better represented by the end point of a regression curve, than by the average of all the data.

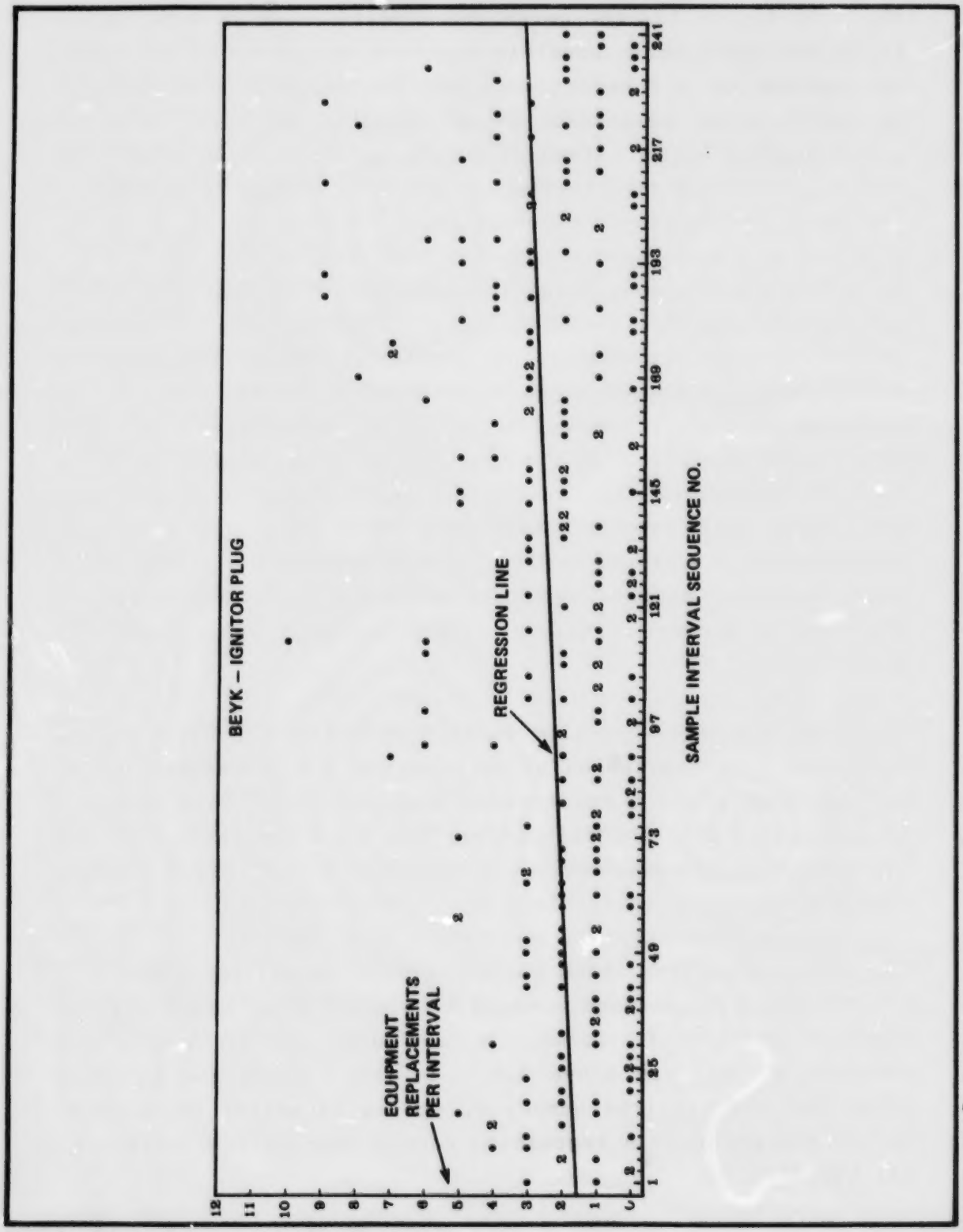


Figure 12: Trend in Frequency of Equipment Replacement

TABLE IV

CHECK FOR TRENDS IN EQUIPMENT REPLACEMENT DATA

NO.	WUC	CORRELATION COEFFICIENT	SIGNIFICANCE LEVEL
1	ABBF	.22	.001
2	AHAA	-.16	.016
3	AHBE	.24	.000
4	AHGL	.02	.759
5	BEKM	.26	.000
6	BEJE	.05	.412
7	BEM	-.25	.000
8	BESA	.04	.586
9	BEYK	.20	.002
10	DAEB	.11	.086
11	EBCB	.04	.533
12	EDA	-.13	.045
13	EJBC	.17	.006
14	EJBE	.23	.000
15	EJBV	-.11	.078
16	FEETB	.03	.661
17	FGEXB	.29	.000
18	FHETA	.13	.047
19	GATTA	-.02	.790
20	GAURA	.11	.087

84. Given a strong trend, a pragmatic approach would be to assume that the Poisson model applies, with V=M, where M is taken to be the end point of a curve fitted to the data. A more mathematically-correct procedure would be to remove the trend by transforming the data. A procedure for performing transformations has been developed by the Directorate of Mathematics and Statistics (DMS) and is documented in Ref. H. The procedure has been tested using CP140 Aurora data and a computer program has been developed to facilitate the analysis.

85. It should be noted that the best fitting regression line will not always be linear. An example of CP140 Aurora data fit with the exponential equation $y = a + be^{-ct}$ is given in Figure 13.* The methodology for fitting non-linear curves to equipment replacement data is also given in Ref. H.

86. Since personnel at CF bases may not have the necessary training or access to computers, trend analysis will be done by DAS Eng 5 staff and the final results will be forwarded to the base.

CYCLIC DATA

87. In addition to long term trends, seasonal variations could be taken into account. For example, the flame holder (BEJE) must be replaced more often in winter than in summer, as demonstrated by the data in Figure 14.* If there are large seasonal variations in the data, one should base the computations on the mean and variance applicable to the relevant season.

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* Acknowledgement: Fig.13 J.A. Dean, Fig.14 Maj. L. Bunce, both of D Log A

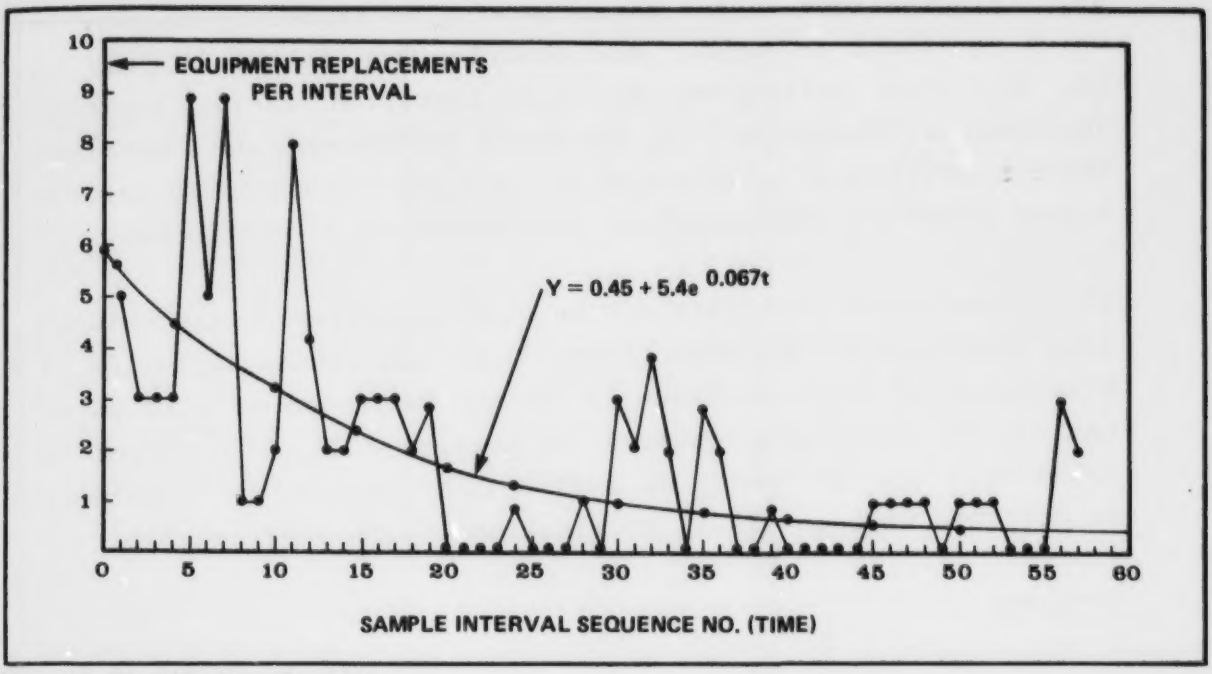

Figure 13: Fit of Non-linear Trend Line

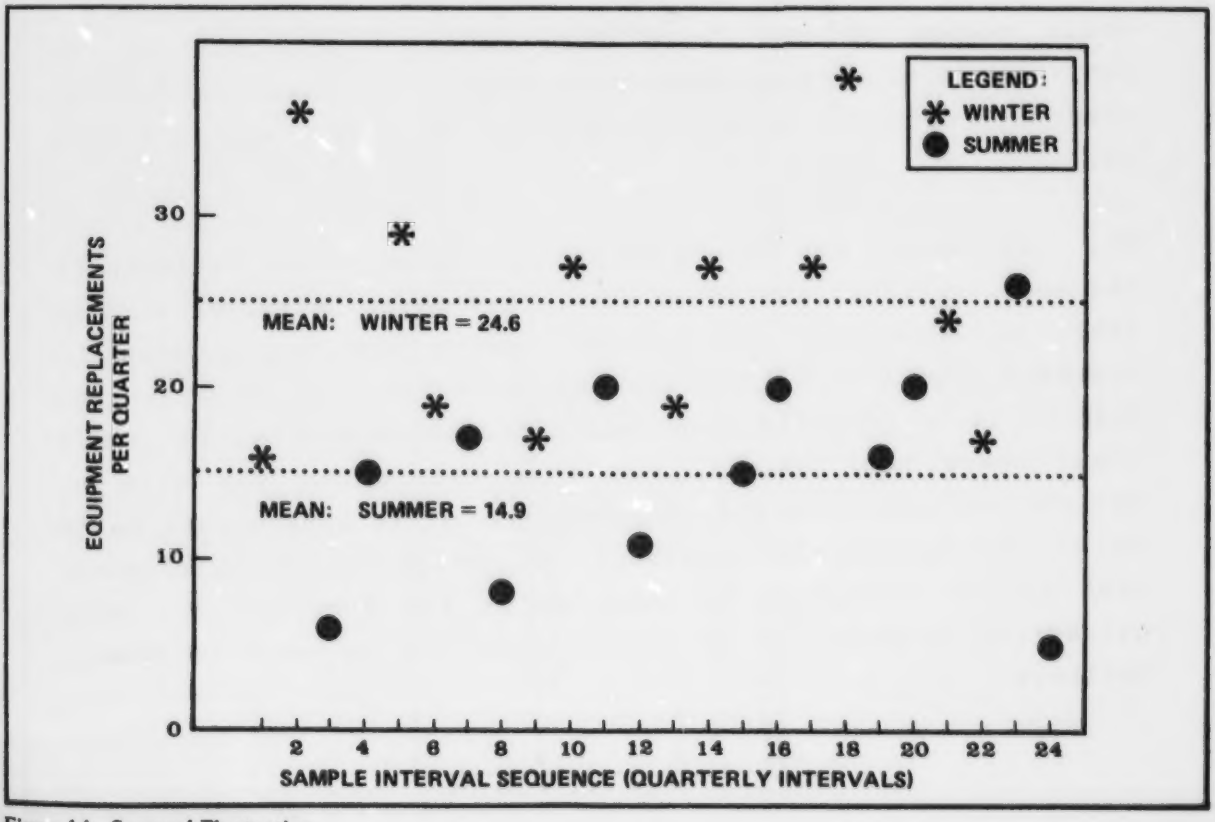

Figure 14: Seasonal Fluctuation

88. Fluctuations in the data can also be caused by the way in which equipment is used. For example, the astral compass in the Sea King helicopter is only installed during northern deployments. Therefore, the equipment replacement data obtained through AMMIS must be adjusted to take into account the periods during which the equipment was not installed or not in use.

89. Time-series analysis can be used to differentiate between long term trends, seasonal fluctuations and random variation. A discussion of this technique is beyond the scope of this paper but it is thoroughly covered in Reference F. It should be noted that the existence of cyclic fluctuations can sometimes be detected through visual examination of the scattergrams.

OUTLIERS

90. The data sometimes contain individual observations that are far removed from the rest. These are called 'outliers'. While review of the CF116 data revealed only one or two outliers, more extreme cases have been encountered in data on other aircraft. An example from CF116 data is given in Figure 15.

91. An outlier can have a strong influence on the estimate of the mean, variance and trend line. This point is illustrated in Table V. Note that the outlier causes the sample mean to increase from 2 to 3.4 and the sample variance to increase from 0.66 to 10.3. It would also create the appearance of an upward trend where none exists. For this reason one may wish to exclude outliers from the computations. It is appropriate to do so if the outlier is considered to be an unusual occurrence that is not likely to be repeated in the future. The data extraction program has an option that can be used to remove outliers.

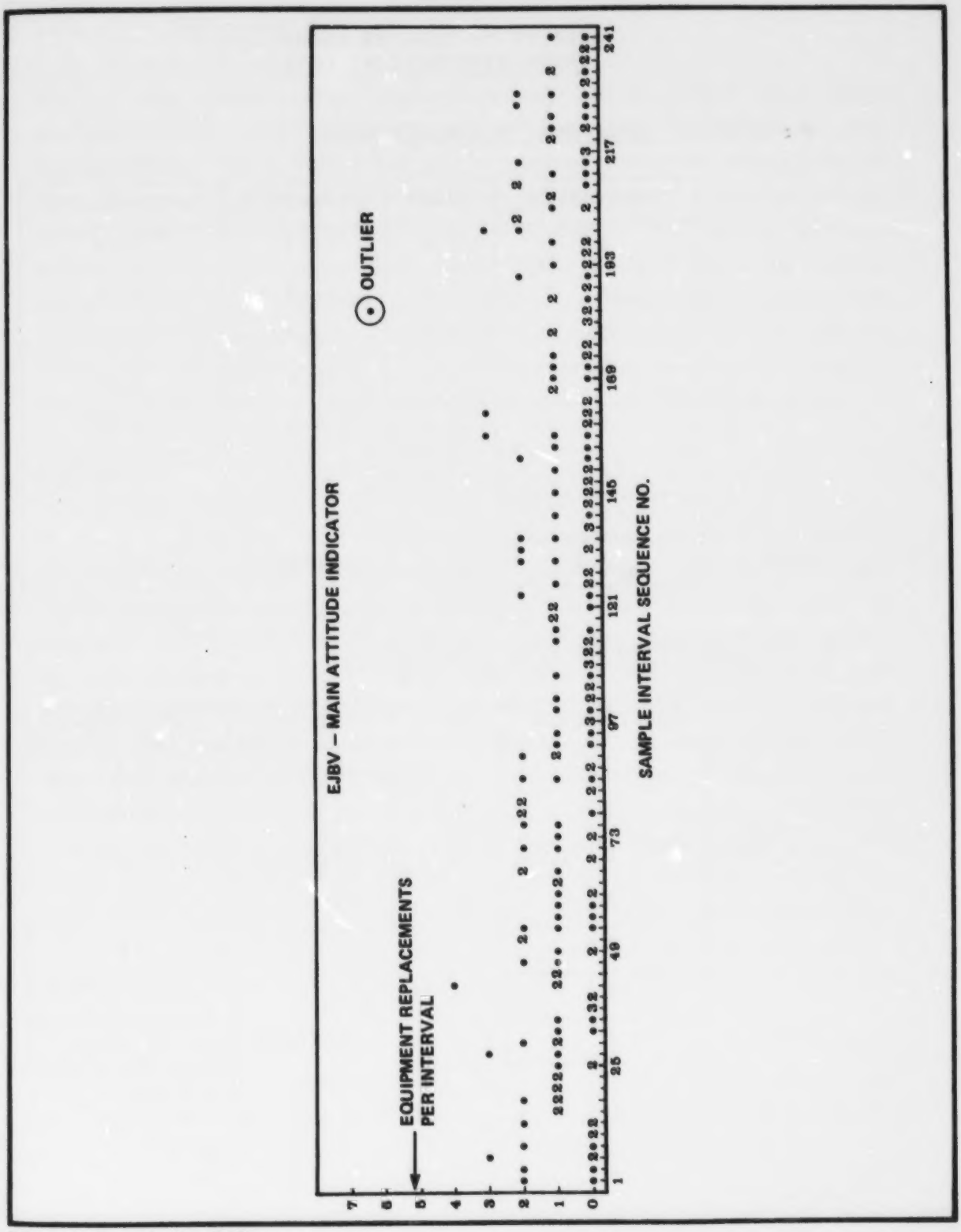

Figure 15: Example of an Outlier

TABLE V

EFFECT OF OUTLIER SHOWN USING HYPOTHETICAL DATA

A. Outlier* Included In Computations

OBSERVATION	MEAN	DEVIATION	(DEVIATION)^2
2	3.4	-1.4	1.96
1	3.4	-2.4	5.76
2	3.4	-1.4	1.96
3	3.4	-0.4	0.16
9*	3.4	5.6	31.36
17			41.20
÷5			÷(5-1)
MEAN 3.4		VARIANCE	10.3

B. Outlier Excluded From Computations

OBSERVATION	MEAN	DEVIATION	(DEVIATION)^2
2	2	0.0	0.0
1	2	-1.0	1.0
2	2	0.0	0.0
3	2	+1.0	1.0
8			2.0
÷4			÷(4-1)
MEAN 2		VARIANCE	0.66

DATA FROM DEPLOYMENTS

92. The observation period during which AMMIS data were gathered may include both operations from home base and deployments. As a consequence, it may not be representative of deployments. Deferred maintenance, changes in environmental conditions, and differences in the extent to which certain equipment are operated may result in a different equipment replacement rate during deployments. Therefore, units are encouraged to keep data regarding the spares used and flying hours for each deployment. Once sufficient actual deployment data is available, it can be used in addition to AMMIS data.

SUMMARY

93. The authors strongly recommend that equipment replacement data be examined before the mean and variance are used in computations. In this way technical expertise can be applied where necessary to ensure that the mathematical model is not misapplied. If the data are problem free and the model's assumptions are appropriate, then this sparing method can be applied with confidence. DAS Eng 5 is willing to check the data and to work closely with users to solve data-related problems.

PART IV - RELAXATION OF ASSUMPTIONS

94. In this part of the report, the original assumptions are re-examined and methodology is discussed regarding alternative assumptions.

REPLACEMENT OF COMPONENT PARTS VERSUS COMPLETE ASSEMBLIES

95. In addition to spare LRUs it may be sensible to include certain component parts in the pack-up kit. The extent to which this is the case depends upon several factors including:

a. the likelihood of such parts being needed; and

b. the usefulness of such parts, given the repair capability, i.e. skills, test equipment, tools and facilities available during the deployment.

In some cases, if component parts are included, fewer LRUs will be needed.

96. AMMIS data can be used to determine the likelihood of such parts being needed. This is illustrated by Table VI which shows the percent of repair actions involving equipment replacement in which a component was replaced rather than an LRU. Note that for the stability augmentation system computer, 96% of the repairs were by component replacement. The canopy balance cable, TACAN and ADF receiver all are close to 50%. For each of these systems, it may be sensible to reduce the number of LRUs in the pack-up kit, replacing them with component spares. AMMIS data can be used to determine the frequency of replacement of the components to be included in the pack-up kit and the frequency of replacement of the LRU less these components.

TABLE VI

PERCENTAGE OF REPAIRS FOR WHICH SPARES WERE REQUIRED IN WHICH PART USED WAS A COMPONENT RATHER THAN A LINE REPLACEABLE UNIT

NO.	WUC	NOMENCLATURE	PERCENT
1	ABBF	CANOPY BALANCE CABLE	52%
2	AHAA	BRAKE ASSEMBLY	5%
3	AHBE	STEERING ACTUATOR	9%
4	AHGL	MAIN UNDERCARRIAGE HYDRAULIC LINE	15%
5	BEKM	VEN LEAF (OUTER)	3%
6	BEJE	FLAME HOLDER	1%
7	BEM	ACCESSORY DRIVE SYSTEM	10%
8	BESA	VEN POWER UNIT	36%
9	BEYK	IGNITOR PLUG (AB)	1%
10	DAEB	CANOPY SEAL	13%
11	EBCB	NOZZLE POSITION INDICATOR	2%
12	EDA	STABILITY AUGMENTATION SYS. COMPUTER	96%
13	EJBC	GHARS ELECTRONIC UNIT	1%
14	EJBE	GHARS CONTROL	8%
15	EJBV	MAIN ATTITUDE INDICATOR	7%
16	FEETB	UHF RECIEVER-TRANSMITTER	1%
17	FGEXB	IFF RECIEVER-TRANSMITTER	2%
18	FHETA	RECEIVER-TRANSMITTER	7%
19	GATTA	TACAN ARN 504	47%
20	GAURA	ADF RECEIVER	46%

REPAIR OF LRUs DURING THE DEPLOYMENT

97. If unserviceable equipment can be repaired during the deployment, then fewer spare LRUs will be required. The number of spares needed will depend not on the duration of the deployment but on the time required to repair each item. Spares would be needed to provide replacement LRUs during the equipment repair time, plus some safety stock.

98. Actual computations in this case involve three considerations:

a. equipment replacement frequency, usually represented by the Poisson distribution;

b. equipment repair time; and

c. allowance for the formation of a queue of unserviceable equipment.

Solutions can be obtained using analytical methods or simulation. However, the methodology required is beyond the scope of this paper. If the problem is simplified by assuming that equipment repair times are constant values and that queues do not form, then a solution can be obtained using the nomographs.

TAKING COST INTO ACCOUNT

99. The preceding work does not take cost into account. This is acceptable if the spares have already been purchased. If they have not, it may be necessary to consider the cost of the items when determining the spares required.

100. The TI59 calculator program described in Reference J provides a method of specifying a pack-up kit which is relatively low cost and which will meet a target kit assurance level that a spare will be available when needed. It is based on a formula which calculates individual item assurance level targets using the item unit cost as a weighting factor. In this way, items with a relatively high unit cost will be assigned a lower target assurance than items with a low unit cost. However, the individual item targets are kept high enough to ensure that the overall kit target will be reached. This program does not require access to a computer and provides good results that can be obtained quickly.

101. The computer program PACMAT documented by Reference K, also takes cost into account when determining the quantity of spares to include in a pack-up kit for a deployment of given duration. It applies a marginal analysis technique in an iterative procedure - in each iteration it selects the item which provides the greatest increase in kit availability per dollar spent. The iterations continue until a target kit availability is achieved or a target budget is reached.

102. A graph is produced which helps maintenance, supply and operational staffs select an appropriate target. The graph is similar to Figure 1, except that the vertical axis is the probability of having enough spares of all items in the kit, i.e. kit availability.

103. PACMAT is very flexible and can accommodate a range of procurement strategies. For example, any desired target budget or target kit availability can be input. Also, minimum and maximum levels can be set on any or all items to constrain procurement. This feature is useful if:

a. certain low-failure-rate items need to be bought for insurance purposes (set MIN = insurance quantity);

b. the quantity of each item in the kit has been determined by some other means and one wishes to calculate kit availability (set MIN = chosen quantity = MAX);

c. procurement in stages is envisioned (set MIN = previously bought quantity, and update mean, variance and cost data) or

d. procurement limits are imposed (set MAX = limit).

104. The computer print-out includes kit availability decrease and increase factors. The achieved kit availability can be multiplied by these numbers to determine the effect of changing the solution by plus or minus one spare of any item.

105. Significant cost savings can be achieved by using either the TI59 calculator program or PACMAT. The former was used to obtain the results presented in Table VII. Note that a cost saving of close to $62,000 or 9% is achieved. Thus cost trade-offs can be beneficial when the pack-up kit is being purchased.

INITIAL PROVISIONING OF REPAIRABLES

106. During initial provisioning of repairables for a new weapon system, one must take into account the failure rate, repair cycle and cost of each item. The optimum allocation of the spares procurement budget can be determined using a computer program called METRIC (Multi-Echelon Technique for Recoverable Item Control,) which is described in Reference L.

TABLE VII
SPARES REQUIRED TAKING COST INTO ACCOUNT

NO.	WUC	ESTIMATED COST	M	V/M	S IGNORING COST	S CONSIDERING COST	DIFFERENCE
1	ABBF	(manufactured by the military)					
2	AHAA	1,047	1.108	1.825	6	9	+3
3	AHBE	2,543	0.357	1.230*	2	3	+1
4	AHGL	47	0.232	1.168*	2	3	+1
5	BEKM	384	2.158	2.861	11	17	+6
6	BEJE	1,466	2.017	1.500	8	10	+2
7	BEM	12,284	2.548	2.148	10	9	-1
8	BESA	4,700	1.722	1.767	7	8	-1
9	BEYK	47	2.170	1.909	9	15	+6
10	DAEB	119	0.129	1.000*	1	3	+2
11	EBCB	108	0.381	0.892*	2	4	+2
12	EDA	5,986	0.502	1.247*	3	3	0
13	EJBC	14,873	2.054	1.668	8	7	-1
14	EJBE	14,747	1.817	1.225*	6	5	-1
15	EJBV	2,024	0.701	1.204	3	4	+1
16	FEETB	4,400	4.191	1.606	12	13	+1
17	FGEXB	11,348	1.971	1.681	8	7	-1
18	FHETA	6,909	0.307	1.075*	2	2	0
19	GATTA	31,632	1.194	1.269*	4	3	-1
20	GAURA	2,584	0.284	1.187*	2	2	0
		Achieved Assurance			0.89	0.90	
		Total Cost			$704,506	$642,449	

* Assumed to be Poisson distributed. The others were assumed to be Compound Poisson.

107. D Log A currently uses an improved version of METRIC known as METRIC-2 and has obtained a more recent version, namely MOD-METRIC.

WAR RESERVE STOCKS

108. In the future, the Canadian Forces may follow the example of other armed forces by purchasing war reserve stocks to support operations in time of emergency or war. The methodology described in this report - nomographs, TI59 calculator program, PACMAT, METRIC - is not directly applicable because it does not take into account the large increase in the number of spares required during the transition from peace-time flying rates to emergency flying rates (see Reference M). The RAND corporation has developed a computer program called DYNA-METRIC to deal with this transition. The USAF later expanded this program into a more powerful model called WARS (Wartime Assessment and Requirements System) (Reference N). Unfortunately, these computer programs are not currently available in DND. Also, a large effort would be required to develop the equipment configuration data and equipment essentiality codes needed to use the models.

109. The METRIC family of computer programs are compared in Table VIII. An overview of spare parts provisioning in the Canadian Forces is given in Reference P.

TABLE VIII

METRIC FAMILY OF
ANALYTICAL SPARING MODELS
FOR REPAIRABLES

CHARACTERISTICS \ MODEL	METRIC	MOD-METRIC	DYNA-METRIC	WARS
PURPOSE	INITIAL PROVISIONING		WAR RESERVE STOCKS	
DATE OF DEVELOPMENT	1967	1973	1980	1981
MEASURE OF EFFECTIVENESS:				
(A) SUPPLY AVAILABILITY, REPAIRABLE ITEMS	Yes	Yes		
(B) OPERATIONAL AVAILABILITY OF WEAPON SYSTEM			Yes	Yes
PERFORMANCE MEASURE VERSUS COST TRADE-OFF	Yes	Yes	Yes	Yes
MULTI-ECHELON (DEPOT, BASE, UNIT)	2	3		3
MULTI-INDENTURE (COMPONENT, ASSEMBLY, MODULE)		2	2	3
MULTI-AIRCRAFT TYPE OR CONFIGURATION				Yes
DYNAMIC PEACE TO WAR TRANSITION			Yes	Yes
SEPARATE PEACETIME/WAR STOCKS				Yes
WAR REQUIREMENTS BY CALENDAR				Yes
MISSION ESSENTIALITY OF PARTS CONSIDERED				Yes
FULL/PARTIAL MISSION CAPABILITY				Yes
LATERAL SUPPORT (UNIT to UNIT)				Yes
CANNIBALIZATION (ROBBING)				Yes
AIRCRAFT ATTRITION (LOST IN BATTLE)				Yes
CONDEMNATION RATE (BATTLE DAMAGE)				Yes
RESUPPLY EXPEDITED OR CUT-OFF				Yes
REPAIR FACILITY CAPACITY LIMITATION				No
BATCH ORDERING OR REPAIR				No

PART V - PUTTING THESE IDEAS TO USE

110. Graphical methods have been provided in this report that can be used to determine the number of spares required to support an aircraft deployment. The methods are based on a quantitative model referred to as the Poisson probability distribution. The formulae and assumptions of Poisson and Compound Poisson models have been described along with examples which show how they can be applied to sparing problems. Also, procedures have been developed to obtain equipment replacement data. The techniques described in this report have been applied to CF116 and CP140 pack-up kits. They are applicable to other Canadian Forces aircraft, and indeed to any equipment maintenance situation where repair is by replacement and repair of failed items is deferred.

111. An effort has been made to anticipate difficulties that will be experienced with the data; also, procedures have been developed to resolve the most common problems - trends, cyclic data and outliers.

112. Inclusion of component parts in the pack-up kit and deployment of a repair capability were discussed. Also, models which take cost into account, including those applicable to initial provisioning and development of war reserve stocks were described.

113. Annex A provides a comprehensive description of the mathematical theory involved in the Poisson model. This Annex may provide the reader with some additional understanding of the theory and confidence in its application. In Annex B, the data extraction method is described in detail. Those interested in applying similar techniques to other maintenance information systems may wish to examine this Annex carefully. Annex D documents the procedure used to create the Compound

Poisson nomographs. This original work might be useful if more Compound Poisson nomographs are required in the future. The procedure should also be applicable in other situations where three-parameter nomographs are required.

114. Although Annexes A, B and D are not required reading, the authors suggest that Annex C is. This Annex provides instructions suitable for personnel at Canadian Forces bases. It explains how to obtain, interpret and use the data. Annex C is intended to be self-explanatory and, although reading the text of this report may be helpful, one should be able to work from the User's Instructions alone.

115. Table IX provides a comparison between a pack-up kit devised using expert opinion without failure data (Ref. Q) and a kit devised using the techniques described in this report. Note that there is a large difference between the two approaches in the probability of having enough spares. The calculations suggest that more ven leaves (outer), GHARS electronic units, GHARS controls and UHF receiver-transmitters should be included in the pack-up kit. It is therefore believed that the nomograph technique can be applied effectively when planning aircraft deployments.

116. The authors recognize that there may be some uncertainty in the minds of potential users, for example:

a. The methods may be considered to be too complicated. Every effort has been made to simplify the procedures as much as possible, yet ensure accuracy in the results and allow technical expertise and judgement to be applied. DAS Eng 5 staff will carry out the data extraction and will assist units by resolving data-related problems.

TABLE IX

CALCULATIONS VS EXPERT OPINION: CF116 SPARES REQUIRED TO SUPPORT 450 FLYING HOURS DEPLOYMENT

NO.	WUC	NOMENCLATURE	CALCULATIONS				OPINION		
			M	V/M	S	P	S	P	
1	ABBF	CANOPY BALANCE CABLE	1.531	2.139	8	0.9933	6	0.9779	
2	AHAA	BRAKE ASSEMBLY	1.108	1.825	6	0.9927	4	0.9674	
3	AHBE	STEERING ACTUATOR	0.357	1.230	2	0.9942	1	0.9496	
4	AHGL	MAIN UNDERCARRIAGE HYD. LINE	0.232	1.168	2	0.9982	2	0.9982	
5	BEKM	VEN LEAF (OUTER)	2.158	2.861	11	0.9920	2	0.6750	
6	BEJE	FLAME HOLDER	2.017	1.500	8	0.9953	4	0.9100	
7	BEM	ACCESSORY DRIVE SYSTEM	2.548	2.148	10	0.9916	7	0.9597	
8	BESA	VEN POWER UNIT	1.722	1.767	7	0.9905	7	0.9905	**
9	BEYK	IGNITOR PLUG (AB)	2.170	1.909	9	0.9937	5	0.9293	
10	DAEB	CANOPY SEAL	0.129	1.000	1	0.9924	4	1.000	
11	EBCD	NOZZLE POSITION IDICATOR	0.381	0.892	2	0.9931	1	0.9435	
12	EDA	SAS COMPUTER	0.502	1.247	3	0.9982	2	0.9855	
13	EJBC	GHARS ELECTRONIC UNIT	2.054	1.668	8	0.9935	2	0.6711	
14	EJBE	GHARS CONTROL	1.817	1.225	6	0.9973	2	0.7261	
15	EJBV	MAIN ATTITUDE INDICATOR	0.701	1.204	3	0.9942	2	0.9657	
16	FEETB	UHF REC-TRANS.	4.191	1.606	12	0.9938	5	0.7301	
17	FGEXB	IFF REC-TRANS.	1.971	1.681	8	0.9942	4	0.9060	
18	FHETA	RECEIVER TRANSMITTER	0.307	1.075	2	0.9962	3	0.9997	
19	GATTA	TACAN ARN 504	1.194	1.269	4	0.9924	5	0.9985	
20	GAURA	ADF RECEIVER	0.284	1.187	2	0.9969	3	0.9998	
PROBABILITY OF HAVING ENOUGH SPARES						0.8899		0.1405	

*S was calculated using a 0.99 item assurance target.

**One spare unit and up to six spare engines with ven power units installed are taken. This is counted as 7 spares. It is faster to change an engine than a ven power unit.

b. Some may argue that the entire procedure could have been 'computerized'. This is true. However, the authors believe that it is essential to involve the unit in the decision-making process. This ensures that authority is not usurped and that knowledge of the operational environment and of the aircraft is fully utilized. Also, units may be required to react quickly to changes in the deployment plan and therefore should not be totally dependent upon computer print-outs from National Defence Headquarters. Finally, the educational impact of widespread use of these procedures may be of great benefit.

c. Some people believe that AMMIS data are not credible. If this were true, it would be unfortunate since AMMIS data are critical to important maintenance management functions, such as the aircraft maintenance rationalizations performed by the Aerospace Maintenance Development Unit (AMDU), and to procurement decisions made by Life Cycle Material Managers (LCMMs). Also, AMMIS data will become more valuable to maintenance personnel at field level when the new system, Automated Data for Aerospace Maintenance (ADAM), is implemented. Therefore, a negative attitude towards AMMIS data is counter-productive. Furthermore, the authors believe that most equipment replacements are recorded.

d. AMMIS data include operations from the base and from deployments. Therefore, its relevance to deployments may be questioned. If deployment planners believe that, for certain items, environmental factors, differences in equipment operation or deferred maintenance should be taken into account, then

appropriate adjustments to the AMMIS data should be made. This would be facilitated by keeping records of spares used during deployments.

e. Some may contend that the number of equipment failures is not proportional to the number of hours flown. It is true that, for some equipment, failure rates are more highly correlated with the number of landings and take-offs or with equipment operating cycles. However, these tend to be correlated with hours flown. Therefore, over a sufficient observation period, the results should be the same.

f. Some argue that aircraft flown intensively are more reliable than aircraft seldom flown, and that this should be taken into account. However, the authors believe that the relationship between flying rate and failure rate is close to directly proportional in the relevant range.

117. The objective of this work was to help Canadian Forces units improve weapon system operational availability. Every effort was made to consider the constraints and difficulties faced during field operations. User's Instructions have been provided for aircraft maintenance supervisors and junior officers. The supporting computer programs are ready to use. DAS Eng 5 awaits requests for data in response to publication of this report.

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Annex A
to ORAE Report No. R 87
dated December 1982

AN INTERPRETATION OF POISSON THEORY

1. A model is a representation of reality. It can be quite simple or very complex. However, to be useful, it must be consistent with observations of the real situation. The Poisson probability distribution has been used effectively as a model of equipment failure patterns. It was used in this report as a method of calculating the number of spares required in a pack-up kit to provide a specified assurance that a spare part will be available if required. The theoretical and practical justification of the Poisson model is discussed in this Annex.

2. To begin with, it must be understood that this discussion will be based on probabilities. In particular, the probability of 0, 1, 2, 3, or more failures of an equipment during an aircraft deployment. The probability that one failure will occur during any particular deployment might be 0.33. This is a theoretical probability statement and can be denoted Prob [X=1] = 0.33. If we observed many deployments and found that 33% of them resulted in one failure, this would be a practical justification of the probability statement. (see Figure A-1).

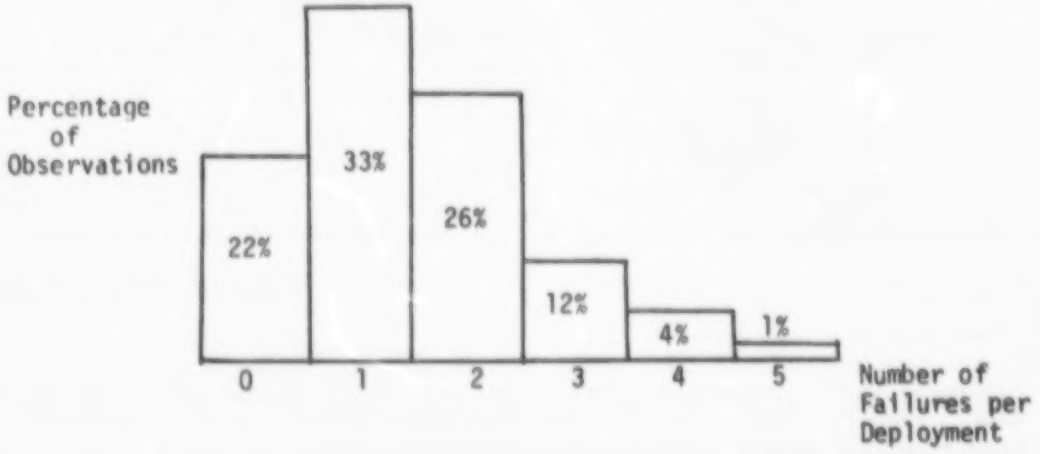


Figure A-1 Expected Observations for Many Deployments

The Poisson Distribution as an Approximation to the Binomial Distribution

3. Consider the situation where an equipment is observed every hour. At each observation, the equipment is either operating or has failed. If it has failed, the observations cease until the equipment is repaired and returned to an operating condition. If these observations were recorded, with successful operation represented by S and failure represented by F, the sequence of observations might be (SSSFSS...SSFSSSSS).

4. Suppose that the observations are independent[1] and the probability of observing a failure is p. Then the probability of observing a success is (1-p). One might ask, what is the probability of observing a given number of failures (x) in the total number of observations (n). First, it can be shown that there are $\frac{n!}{(n-x)!\,x!}$ sequences of the form above which have x failures and (n-x) successes. The combinations symbol $\binom{n}{x}$ is often used as notational shorthand for this expression. Each of these sequences occur with probability $p^x(1-p)^{n-x}$. Thus the probability of exactly x failures in n observations is

$$\text{Prob}\left[X = x\right] = \binom{n}{x} p^x (1-p)^{n-x}$$

The probability of x or less failures in n observations is

$$\text{Prob}\left[X \le x\right] = \binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1} + \ldots + \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$$

The formula above is referred to as the Binomial distribution.

---

1. 'Independent observations' implies that knowledge concerning the result of one observation will not provide any information about subsequent observations.

5. If n is large and p is small, the Binomial distribution can be approximated using the Poisson distribution with the parameter $\lambda t = np$. The Poisson distribution can be expressed by the formulae:

$$\text{Prob}[X=x] = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

and

$$\text{Prob}[X \leq x] = \sum_{i=0}^{x} \frac{(\lambda t)^i e^{-\lambda t}}{i!}$$

It will be shown later that λt is the failure rate per interval.

6. For example, if an equipment was observed for 100 operating hours and it was known that the probability of failure in any given hour was .01, then the Poisson distribution with parameter $\lambda t = np = 1.0$ could be used as an approximation to the Binomial distribution (see Table A-I).

TABLE A-I

AN EXAMPLE OF THE POISSON APPROXIMATION

TO THE BINOMIAL DISTRIBUTION WITH $\lambda t = np = 1.0$

x	Binomial Prob [X=x]	Poisson Prob [X=x]
0	0.3660	0.3679
1	0.3697	0.3679
2	0.1849	0.1839
3	0.0610	0.0613
4	0.0149	0.0153
5	0.0029	0.0030
6	0.0004	0.0005
7	0.0001	0.0001

The Poisson Process

7. In the previous section, the Poisson distribution was introduced as an approximation to the Binomial distribution. However, using an alternative formulation of the same problem, it can be shown that the Poisson distribution can stand on its own.

8. Consider the situation where an equipment begins running and runs until failure. The failure is then repaired by the replacement of one part and the equipment continues running. The number of failures (or parts used) can be described as a function of equipment operating time, denoted N(t). This function might look similar to Figure A-2.

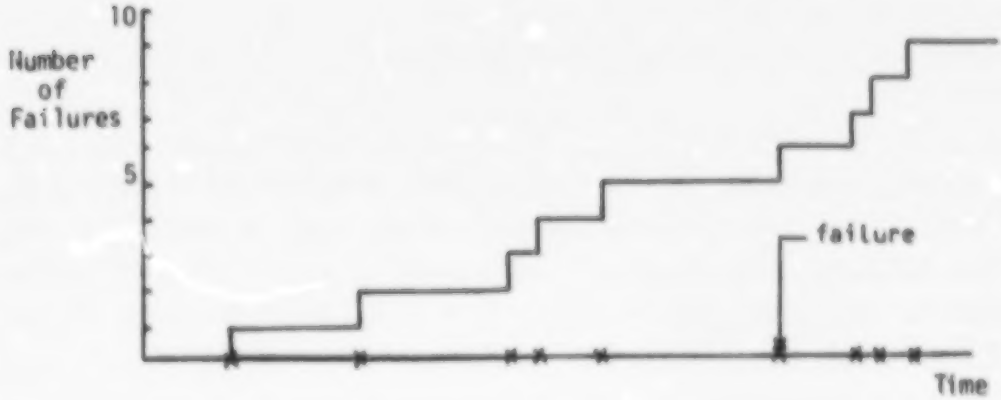


FIGURE A-2 The Poisson Counting Process

9. If the function N(t) satisfies the following four assumptions, it is referred to as a Poisson counting process:

a. The counting process starts with the number of failures equal to zero;

b. The probability that a failure will occur in a small interval of time is essentially proportional to the length of the interval;

c. The probability of more than one failure in a small interval of time is very small; and

d. The number of failures in non-overlapping, equal intervals of time is independent.

10. Assumption a. is simply an initial condition (i.e. $N(0)=0$). In assumption b., the 'proportion' is described by the Poisson parameter λ. It can be thought of as the number of failures in one unit time or failure rate. Thus, in probability terminology, assumption b. implies:

$$\text{Prob}[N(t+\Delta t) - N(t)=1] \approx \lambda\Delta t \text{ ; and}$$

assumption c. implies

$$\text{Prob}[N(t+\Delta t) - N(t)>1] \approx 0.$$

Thus

$$\text{Prob}[N(t+\Delta t) - N(t)=0] \approx 1-\lambda\Delta t.$$

It should be noted that these formulae are assumed to apply only for a small interval around t. However, they are used to develop more useful general formulae for larger intervals.

11. Assumption d. is the most important one because it expresses the 'memoryless' nature of the Poisson process. That is, failures in previous intervals do not affect the expected number of failures in the current interval. So, if $t=t_2-t_1$ then, the expected number of failures in the interval (t_1,t_2) is the same as the expected number of failures in $(0,t)$ (see Figure A-3). The expected number may be fractional, such as 0.333, while in practice the number of failures per interval must be either 0,1,2, ... However, if enough observations were made, the average number of failures per interval would be close to the theoretically expected number.

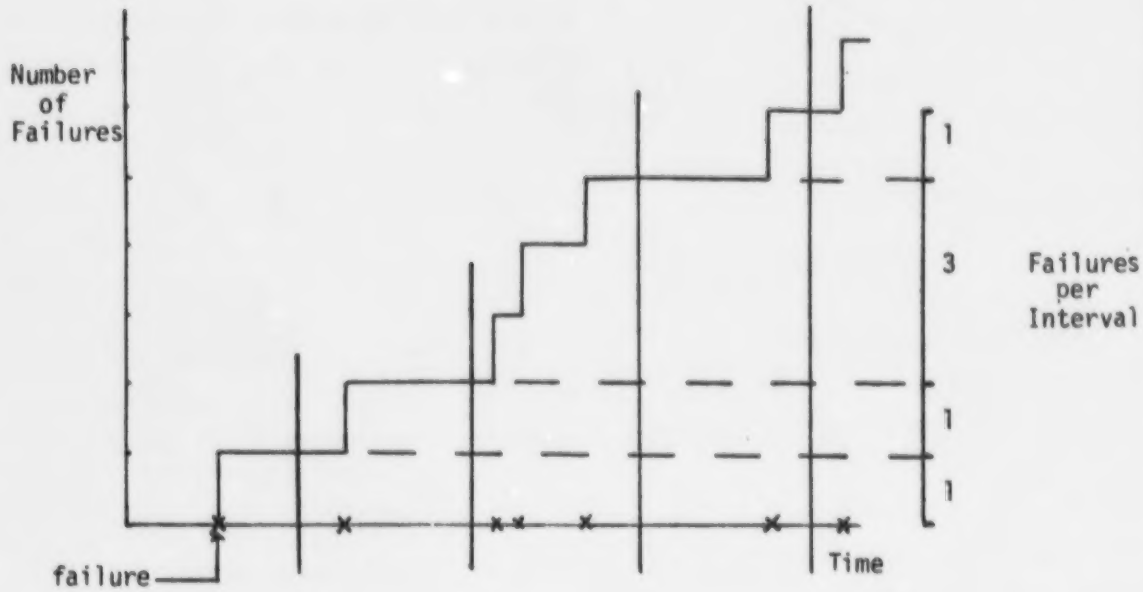


Figure A-3 Expected Failures in an Interval of Time

12. Based on these four assumptions, it can be shown that the number of failures occurring in any given interval of time has a Poisson probability distribution (Ref.1, p.454-460). That is, for $t=t_2-t_1$,

$$\text{Prob}\left[N(t_2) - N(t_1)=x\right] = \text{Prob}\left[N(t)=x\right] = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

and

$$\text{Prob}\left[N(t)\leq x\right] = \text{Prob}\left[N(t)=0\right] + \text{Prob}\left[N(t)=1\right] \ldots \text{Prob}\left[N(t)=x\right]$$

$$= \sum_{i=0}^{x} \frac{(\lambda t)^i e^{-\lambda t}}{i!}$$

This probability function can be portrayed graphically as shown in Figure A-4. Notice that for small values of λt, the Poisson probability function is skewed to the right.

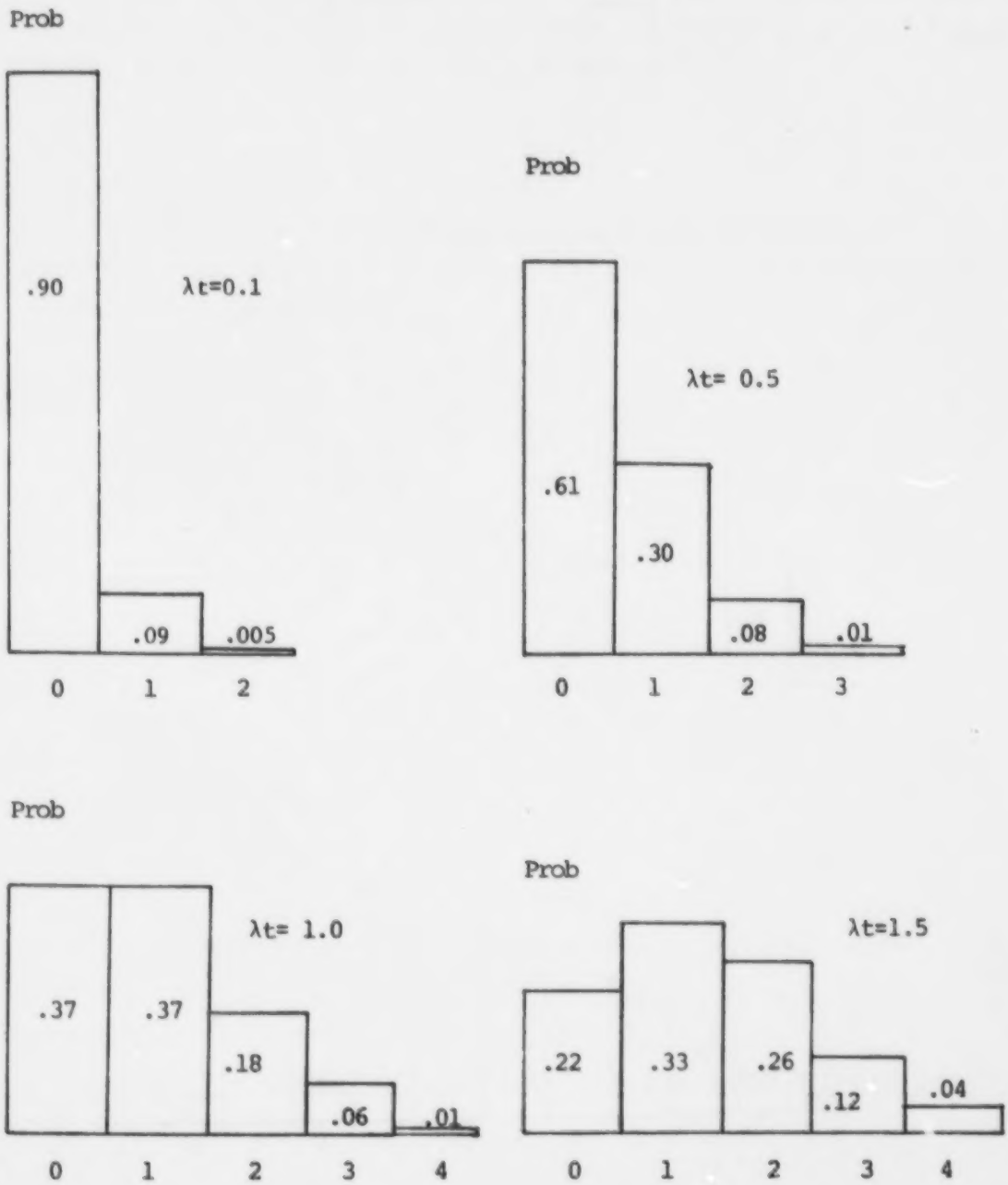


FIGURE A-4 Examples of Poisson Probability Distribution

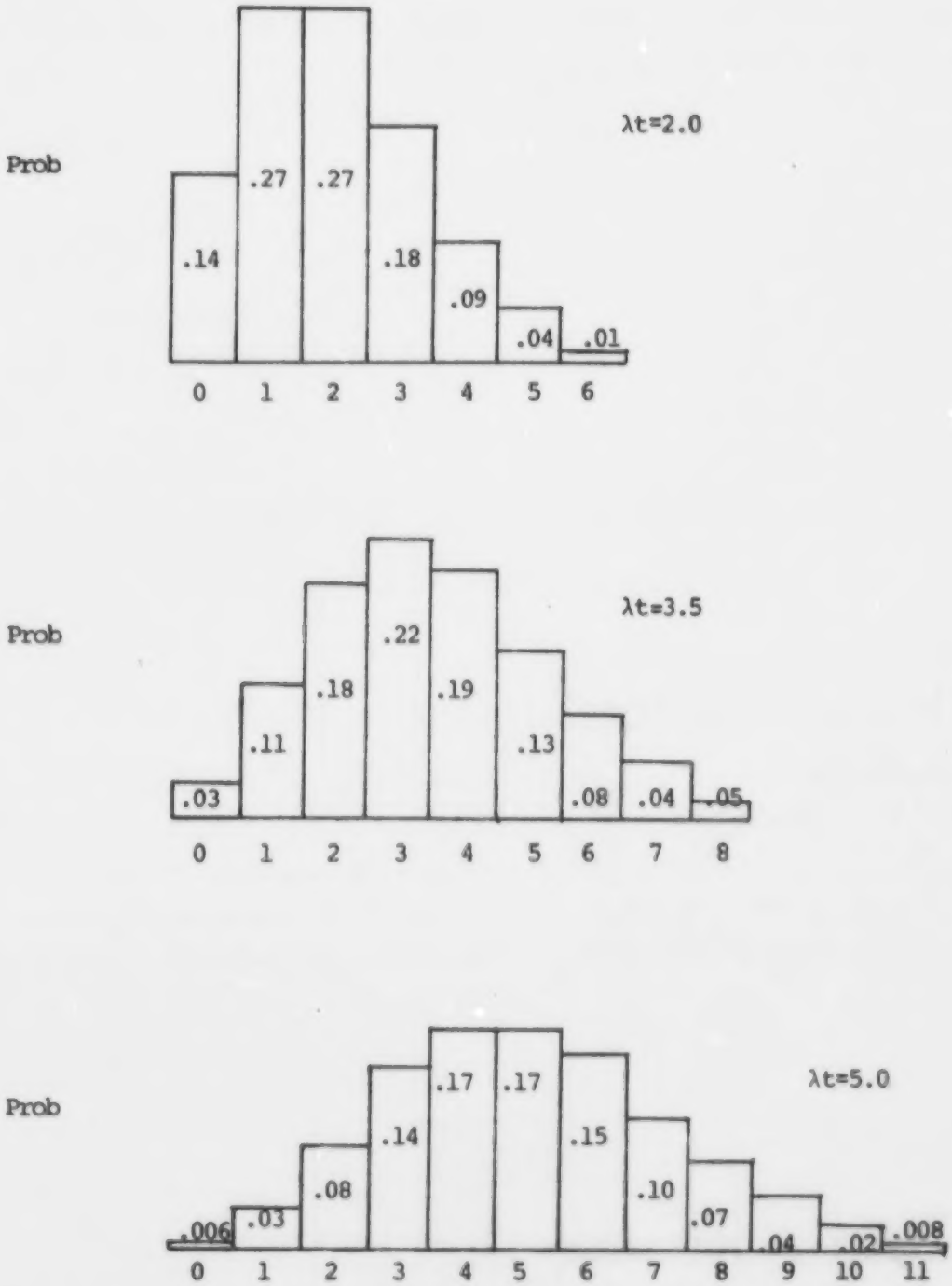


Figure A-4 (Continued) Examples of Poisson Probability Distributions

13. Another method of justifying the Poisson process involves the analysis of failure times. Consider Figure A-5. If the values of N(t) are distributed according to a Poisson distribution with parameter λt, the values of time between failure (Z_k) are independent and exponentially distributed (Ref. 1, p.469). That is, for all k=0,1,...

$$\text{Prob}\left[Z_k \leq t \right] = 1 - e^{-\lambda t}$$

Using the reverse logic, if the time between failure can be shown to be exponential, then the number of failures before time t will be Poisson.

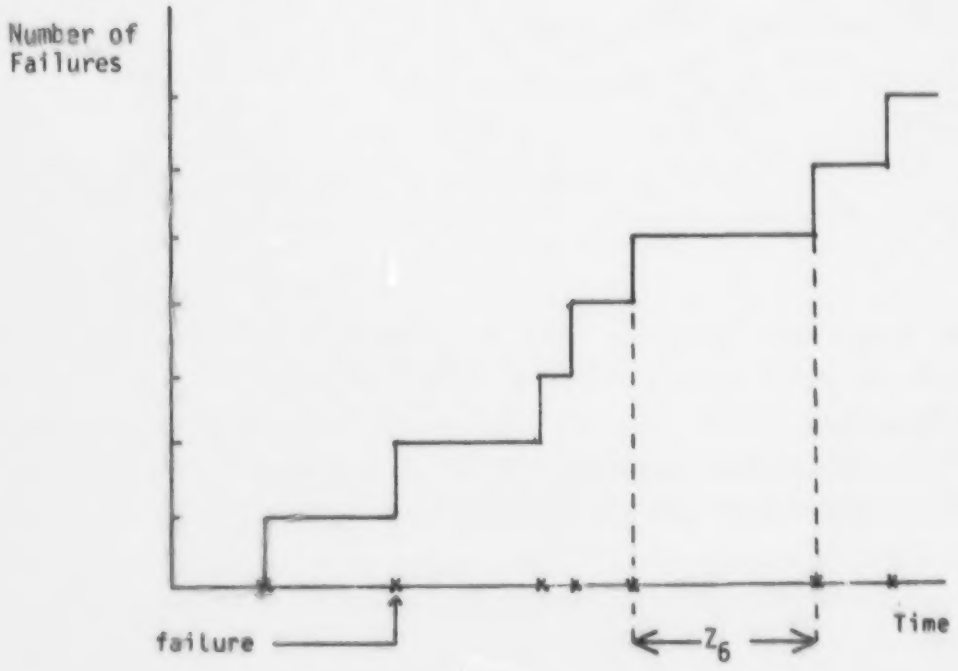


Figure A-5 Exponential Distribution for Time Between Failure

14. If N(t) is a Poisson counting process with parameter λt, the expected value (or mean) of N(t) given t can be found in general and is denoted

$$E[N(t)] = \sum_{x=0}^{\infty} x \cdot \text{Prob}\,[N(t) = x]$$

$$= \lambda t \quad \text{(Ref. 2, p. 224)}.$$

Thus, the expected number of failures in a given interval of time is a function of the length of the interval. The variance is a theoretical measure of dispersion. It can be calculated for a given t, as follows

$$\text{Var}\,[N(t)] = E[N(t)^2] - E[N(t)]^2$$

$$= \sum_{x=0}^{\infty} x^2 \text{Prob}[N(t)=x] - (\lambda t)^2$$

$$= (\lambda t)^2 + (\lambda t) - (\lambda t)^2$$

$$= \lambda t \quad \text{(Ref. 2, p. 228)}.$$

Notice that the variance for a given t is also a linear function of the length of the interval and is equal to the expected value. That is, as the time interval increases in size, the expected number of failures increases proportionally as does the variance (see Figure A-4).

15. The first result provides an interpretation of the parameter λ. Since $E[N(t)] = \lambda t$,

then $\lambda = \dfrac{E[N(t)]}{t}$.

That is, λ is the expected number of failures per unit time. Now if $X_1, X_2, \ldots, X_n$ are observations of the number of failures in a given time interval, then the arithmetic mean provides a good estimate of $E[N(t)]$, that is

$$E[N(t)] \sim \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

and

$$\lambda \sim \frac{\bar{X}}{t} \quad .$$

In other words, λ can be approximated by the average number of failures divided by the length of the interval observed. A more commonly used term is mean time between failure (MTBF) which can be calculated by dividing the time interval by the average number of failures per interval. That is,

$$MTBF = \frac{t}{\bar{X}} \approx \frac{1}{\lambda}$$

16. The second result provides a 'quick and dirty' test of the validity of the Poisson distribution. The sample variance

$$S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

provides as an estimate of Var [N(t)]. If the sample variance S^2 divided by the sample mean $\bar{X}$ is approximately equal to one, the Poisson distribution should produce reasonable results. If this ratio is greater than one, the Poisson distribution may not apply[1]. The Compound Poisson distribution described in the next section would be more appropriate.

---

1 In order to test how closely practical experience would follow this theoretical result, a computer program was developed which used random numbers to create samples corresponding to a Poisson distribution. Analysis of these samples indicated that the variance to mean ratio was in most cases between 0.7 and 1.2. Therefore, in practice, the Poisson distribution may be applied when the variance to mean ratio is not greater than 1.2.

Compound Poisson Distribution

17. Consider the situation at a supply facility where customers requiring a certain item arrive according to a Poisson process. However, the customers may request 1, 2, 3 or more of the particular item. This is referred to as a Compound Poisson process and can be depicted as shown in Figure A-6. Here N(t) refers to the number of items requested from supply by time t.

Figure A-6 Compound Poisson Process

18. In practice, one of the following situations might lead to a Compound Poisson distribution:

a. sympathetic replacement, where a maintainer, upon discovery of a defective item on one aircraft, inspects other aircraft and replaces that item to avoid future failures;

b. burn-in failures, where some components have a high probability of failure shortly after installation; or

c. highly correlated flying programs, where two or more aircraft can have the same type of failure at the same time (Ref.3, p.6).

These situations would lead to 'bunching' in demand rate observations. It is not important to know why the 'bunching' occurred. However, if 'bunching' does occur, it is important to use the Compound Poisson process rather than the Poisson process to take this into account when calculating the spares required.

19. Compound Poisson formulae can be written in the form

$$\text{Prob}\left[N(t)=x\right]=\left|\sum_{y=1}^{\infty}\frac{(\lambda t)^{y}e^{-\lambda t}}{y!}\right| f(y,x)$$

The first term is the probability that y customers arrive in a given time t and f(y,x) is the probability that y customers will request x items. It can be shown that

$$E\left[N(t)\right] = (\lambda t)E[W]$$

where E[W] is the expected number of requests per customer and λt is the expected number of arrivals in time t.

Furthermore,

$$\text{Var}[N(t)] = (\lambda t)\, E[W^2]$$

Thus the variance to mean ratio,

$$R = \frac{\text{Var}[N(t)]}{E[N(t)]} = \frac{\lambda t\, E[W^2]}{\lambda t\, E[W]} = \frac{E[W^2]}{E[W]}$$

is independent of the customer arrival rate and independent of the time interval (Ref.4, p.190).

20. Much of the literature refers to the Negative Binomial distribution which can be shown to be a Compound Poisson distribution (Refs. 2,3,4). For the Negative Binomial distribution, one can express the probability of 1,2,3 or more demands per arrival using a Logarithmic distribution (Ref. 2, p. 291). This probability is a function of the variance to mean ratio (R). As shown in Figure A-7, the probability that the demands per arrival is greater than 1 becomes larger as R increases.

21. Instead of using the compound formula as shown in paragraph 19, the following formulae are used to perform the computations:

$$\text{Prob}\,[N(t) = 0] = \left(\frac{1}{R}\right)^{\frac{M}{R-1}}$$

and for x = 0, 1, 2, ...

$$\text{Prob}\,[N(t) = x+1] = \binom{x+M}{x+1}\left(\frac{R-1}{R}\right)\text{Prob}\,[N(t)=x]$$

where M=the mean; and

R=the variance to mean ratio.

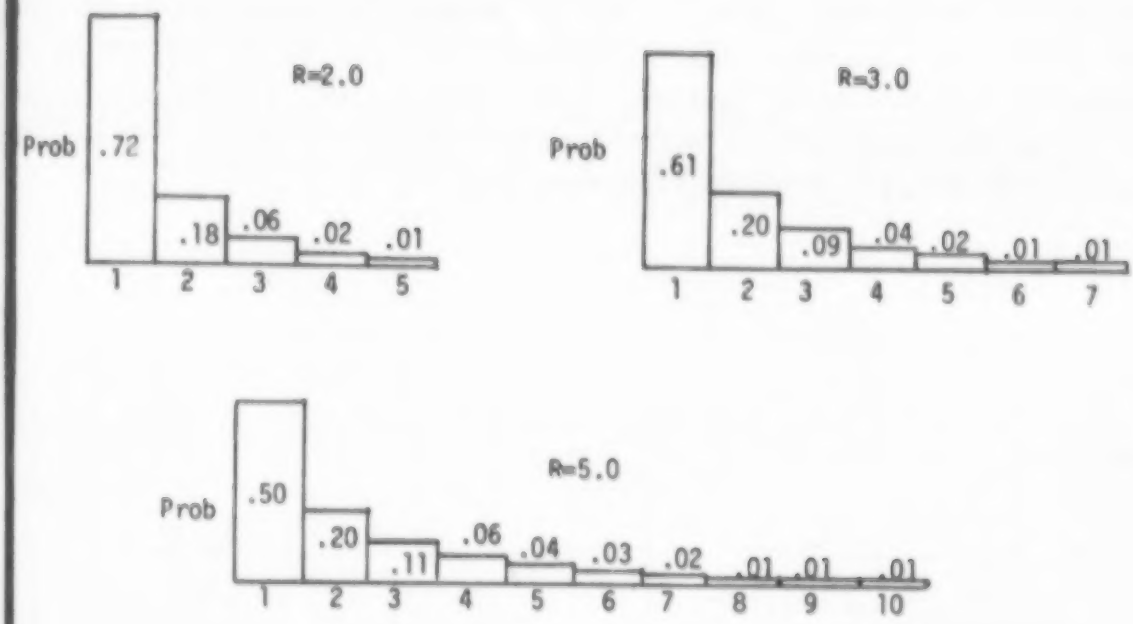


Figure A-7 Probability of One or More Demands Per Customer Arrival (Logarithmic Distribution)

Figure A-8 gives examples of the Negative Binomial distribution for various values of M and R. It can be seen that, with R constant, as M increases the distribution becomes less skewed and more symmetric. Also it can be seen that with M constant, as R increases the tail of the distribution becomes extended.

22. In practice, the sample mean and sample variance can be calculated. If the variance to mean ratio is greater than one, the Compound Poisson distribution can be used instead of the Poisson distribution to calculate the spares required to meet a specified assurance level. The Compound Poisson distribution will take into account the 'bunching' in the data. By using a Compound Poisson distribution, the number of spares required for a given mean and a given assurance level will be higher than if the Poisson distribution were used.

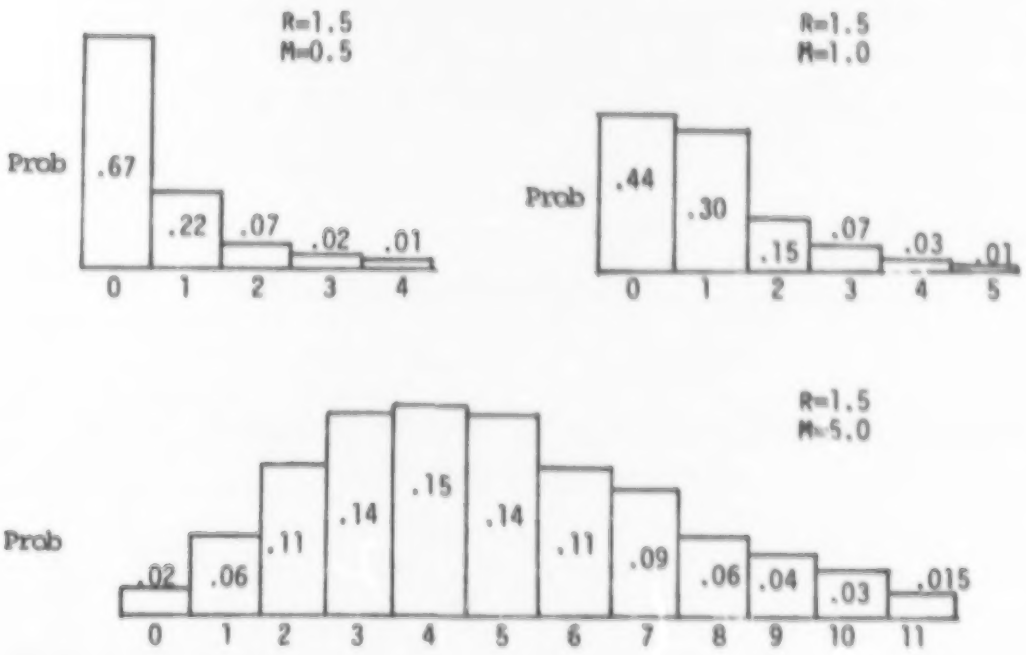


Figure A-8 Examples of Compound Poisson Probability Distributions

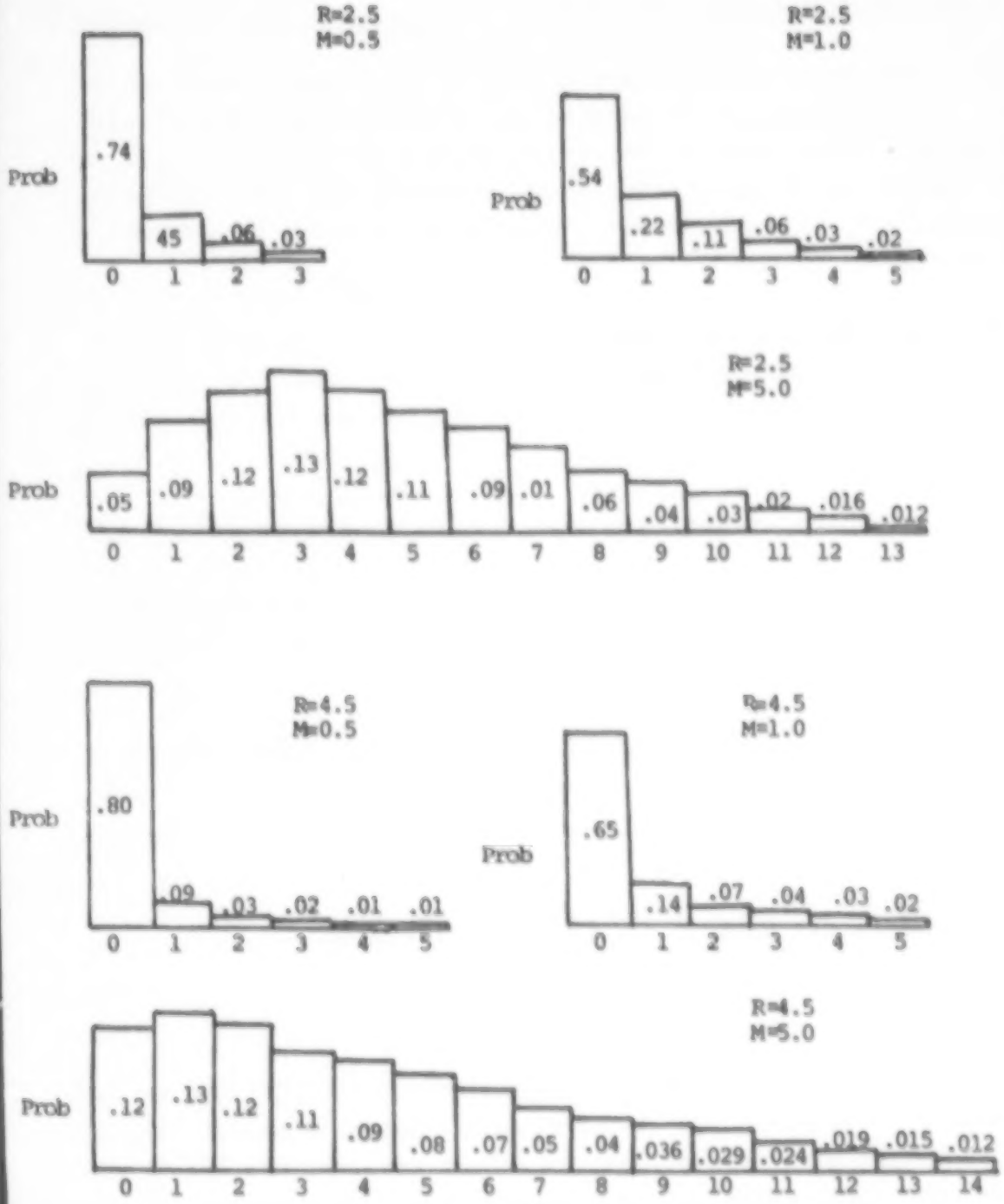


Figure A-8 (Continued) Examples of Compound Poisson Probability Distributions

Using the Models with Confidence

23. Before applying a particular probability distribution, it is often worthwhile to examine the distribution of the data. When examining demands for spare parts, one should develop a histogram which shows the values observed and the number of occurrences of each value. Example histograms are shown in Figure A-9.

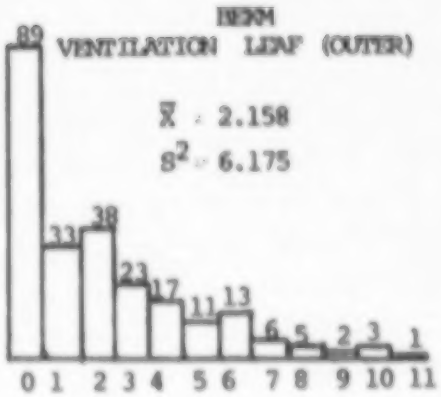
24. To evaluate the goodness-of-fit between the data and the theoretical distribution, the Chi-Squared test is commonly used. However, there are two practical problems with the use of this test:

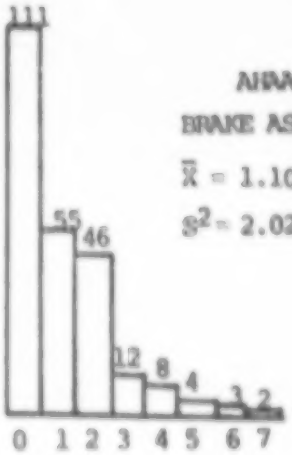
a. The procedure requires the histogram to be grouped into cells of at least five observations each. The skewed nature of the Poisson distribution can make it difficult to obtain a sufficient number of cells to epply the test to demand data.

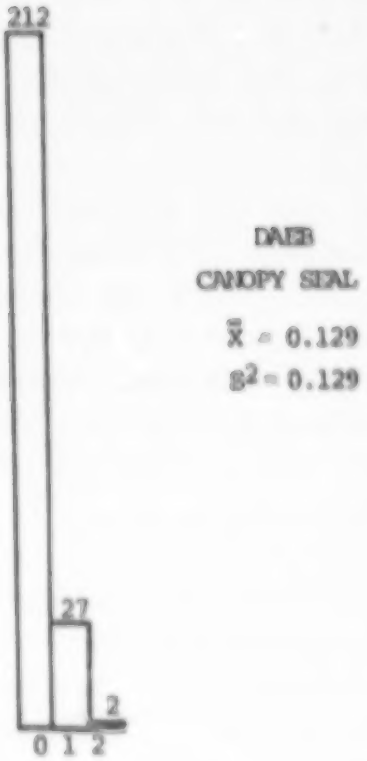
b. Because of this grouping, the test does not apply well to the tails of the distribution. In most sparing studies, the tail of the distribution is used.

Although the Chi-Squared test provides a rigorous justification of the model, in practice, it is difficult to apply to demand data.

25. Thus, it is recommended that a histogram of the data be visually inspected to evaluate the goodness-of-fit to a Poisson-type distribution. If the actual observations appear to have the same shape as the theoretical distribution, (i.e. unimodal, skewed to the right, with smooth transitions) then that model may be used with confidence.







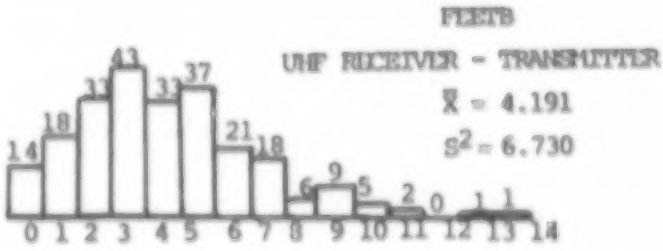


Figure A-9 Example Histograms for CF116 Data

Data Requirements

26. Data concerning the parts used in actual deployments would provide the best results in the study of pack-up kit requirements. However, these data are not available and will probably not be available in a large enough quantity to be useful in the near future. Equipment replacement data are available from the Aircraft Maintenance Management Information System (AMMIS). However, for this data to be useful in pack-up kit studies, it is important that it be representative of a deployment. Thus, the flying hours per interval should be close to the expected flying hours per deployment. Also, periods with high activity rates may be more representative of deployments than those with low activity rates.

Monitoring Deployments

27. If the sparing analysis of pack-up kits is considered important in the future, it may be worthwhile to collect parts usage information from the deployments themselves. As a minimum, it will be necessary to validate the use of AMMIS data by comparing the results of actual deployments with the expected results based on AMMIS data.

28. There are three problems that may occur:

a. not enough of some parts;

b. too many of some parts; and

c. parts required that are not included in the kit.

29. It is hoped that the first problem will not occur too often using the Poisson sparing method. However, some shortages are to be expected. For example, if the kit is spared to a 0.90 level, then one out of every ten deployments will be expected to suffer a shortage. If the shortages are excessive, then the replacement rates obtained from AMMIS data may not be representative of deployments. Methods might be developed to adjust the replacement rates as required.

30. The second problem is not serious unless weight, cost or criticality factors must be considered. To meet a specified assurance level, the pack-up kit must contain a certain amount of safety stock. If one wishes to obtain minimum safety stock to meet the requirement, an optimization technique might be applied which incorporates weight, cost or criticality factors.

31. This safety stock idea can be visualized through an example. Consider a pack-up kit with 100 items in which each item has an expected demand rate of 1.0 per deployment. Using the Poisson probability distribution, the expected results of a deployment are shown in Table A-II for a pack-up kit containing 0, 1, 2, 3 and 4 of each item. Notice that even with 3 spares for each item in the kit, the probability of having a deployment without a shortage is $(.9810)^{100} = .1469$ and the expected results would suggest that there will be a shortage in two items. To obtain a 70% assurance level for the kit as a whole there would be 300 surplus items (see Table A-II). Table A-III presents the same idea applied to 100 items with an average demand of 0.5 per deployment. For most items in a typical kit, the average demand rate will be less than one per deployment. However, to be assured of having a part if needed surpluses in some items will be expected.

TABLE A-II

EXPECTED RESULTS WITH 100 ITEMS EACH
WITH AN AVERAGE DEMAND OF 1 PER DEPLOYMENT

Number of Spares per Item	Assurance Level		Expected Results per Deployment		
	Item	Kit	Shortage	Consumption	Surplus
0	.3679	.0000	64	0	0
1	.7358	.0000	26	64	36
2	.9197	.0002	8	90	110
3	.9810	.1469	2	98	202
4	.9963	.6903	0	100	300

TABLE A-III

EXPECTED RESULTS WITH 100 ITEMS EACH WITH
AN AVERAGE DEMAND OF .5 PER DEPLOYMENT

Number of Spares per Item	Assurance Level		Expected Results per Deployment		
	Item	Kit	Shortage	Consumption	Surplus
0	.6065	.0000	39	0	0
1	.9098	.0001	9	39	61
2	.9856	.2345	2	48	152
3	.9982	.8351	0	50	250

32. The third problem involves the choice of items that should be put in a pack-up kit. Items with high demand rates will need to be included in fairly large number as shown in the previous discussion. Items with low demand rates may also be included. In fact, a RAND study found that the items with the lowest demand rates cause the largest numbers of shortages (Ref. 5). This is because there are so many more items at these low demand rates, most of which are not even considered for inclusion in the kit.

33. In a modern aircraft, there may be in the order of 15,000 components. About half of these will have a replacement rate less than one every three years. Thus in any month, the probability of replacing a low failure rate item is $1/36=.028$. However, the probability of not replacing any of these items is $(1-.028)^{7500} \simeq 0.0000$. Thus, the question of what to include in a pack-up kit requires good judgement as well as knowledge of the item replacement rates. It is hoped that the methods described in this annex will allow the expertise and good judgement already existing among operational planners to be applied to these difficult problems.

ADDITIONAL REFERENCES FOR ANNEX A

1. Davenport, W.B., Probability and Random Process: An Introduction for Applied Scientists and Engineers; McGraw-Hill, New York, 1970.

2. Feller, W., An Introduction to Probability Theory and Its Applications: Volume I; John Wiley and Sons, Third Edition, 1968.

3. Feeney, G.J., Sherbrooke, C.C., The (S-1, S) Inventory Policy Under Compound Poisson Demand: A Theory of Recoverable Item Stockage; The RAND Corporation, March 1966(AD 672-431).

4. Sherbrooke, C.C., Discrete Compound Poisson Processes and Tables of the Geometric Poisson Distribution; Naval Research Logistics Quarterly, Vol. 15, No.2, June 1968 (AD 672-664).

5. Karr, H.W., Geisler, M.A., Brown, B.B., A Preferred Method for Designing a Flyaway Kit; The RAND Corporation, May 1955.

ANNEX B
to ORAE Report No. R 87
dated December 1982

DATA EXTRACTION

Purpose

1. The data required to calculate the quantity of spares needed to support an aircraft deployment can be obtained from the Aircraft Maintenance Management Information System (AMMIS). The data extraction procedures are described in this Annex. These procedures were developed using a 434 Squadron deployment of CF116 Freedom Fighter aircraft as an example. However, the method also has been applied to CP140 Aurora deployments and should apply equally well to any aircraft type.

The Function of AMMIS

2. AMMIS collects data regarding aircraft maintenance. Data summaries are provided in three forms:

a. routine reports published on a monthly basis;

b. ad-hoc data retrievals using standard procedures; and

c. special purpose reports generated in response to specific requests.

Why Not CFSS?

3. The Canadian Forces Supply System (CFSS) provides information regarding parts usage. However, the supply organization is not necessarily informed about items

returned to base shops for repair. Therefore, CFSS is not, in general, an acceptable source of data for the purpose of calculating the number of spares required for pack-up kits, although it may be appropriate for consumables or items that are sent to civilian contractor for repair.

Data Extraction

4. For each item in the pack-up kit, data were needed on the frequency of equipment replacement. The requirement was met by extracting data on equipment failures from AMMIS and data on aircraft flying hours from the Aircraft Utilization Statistical Report (AUSR).

5. Six years of maintenance history were divided into intervals of equal flying hours. The number of equipment replacements during each interval was then determined using AMMIS data. The interval length chosen was representative of a typical deployment. Thus, the fleet operations were looked upon as a series of back-to-back deployments. Using an interval length of 450 flying hours, 241 observations were obtained. For future computer runs, the interval length can be set at any value desired.

6. To identify instances of equipment replacement, a computer search of aircraft maintenance records was performed. The first step was to select all repair actions coded as'FIX 3' which covers adjustments, repairs and replacements. In the second step, replacements were isolated using the code for equipment disposal. A code of 'DISP 6' indicates that equipment had been removed and either repaired locally or shipped to a contractor for repair. A code of 'DISP 8' indicates that equipment had been removed and scrapped. Thus the code 'FIX 3' combined with 'DISP 6' or 'DISP 8' indicates that a spare was required.

7. The following categories of maintenance actions were considered relevant:

a. FOD - failures that were caused by Foreign Object Damage. Spares would be required in this case.

b. Found on a Major Inspection - failures that were corrected while the aircraft or the non-installed engine was on a major inspection. It is assumed that the equipment would have failed during operation if the failure had not been detected and corrected during the inspection.

c. No Fault Found in Shop - items that were removed from the aircraft but the stated unserviceability was not confirmed in shop. Assuming no shop support during the deployment, the fact that the equipment is actually serviceable will not be known until it is checked in the shop upon completion of the deployment.

8. The following categories of maintenance actions were excluded:

a. Scheduled Removals - items scheduled to be removed because they are 'time-expired'. Scheduled removals that will become due during a deployment are usually removed prior to the deployment. In those instances where this is not the case, it is a simple matter to add quantities required for scheduled removals.

b. No Fault Found on Aircraft - items for which the stated unserviceability was not confirmed while the equipment was still installed in the aircraft. No spare would be required in this case.

9. It was assumed that any differences amongst squadrons in type of operation or operating environment did not significantly affect the equipment failure rates. Thus, data from all bases flying a given aircraft type were included. This approach provided a large sample for statistical analysis. However, data could be extracted for a particular region or unit if that seemed to be more appropriate.

Other Considerations

10. Some other points merit discussion:

a. Equipment replacement vs repair.

It is assumed that during deployments all failures are rectified by replacing faulty equipment. That is, one spare line replaceable unit is required for each failure that occurs. This is not the case during operations from the base where repair by replacement of component parts is possible. Therefore, it is necessary to obtain two sets of data:

(1) failures rectified by replacing the faulty equipment; and

(2) failures rectified by replacing component parts.

If (2) is a large percentage of total repairs, then the proposed method of repair during deployment should be investigated and whichever data is appropriate, (1) or (2), should be selected for use in the calculations.

b. More than one aircraft model

Some fleets are comprised of more than one aircraft model. The models differ in the quantity of installed equipment for certain systems. For example, the CF116 fleet is comprised of single and dual seat aircraft in which one ADF receiver is installed in the singles but none in the duals. For this reason, the frequency of equipment replacement was determined separately for each model.

c. Multiple installations

Some items are installed in more than one location in the aircraft. A different work unit code may be assigned to each location. The computer program has an option that can be used to merge data extracted against these different work unit codes.

d. Changes in equipment reliability

The data can be evaluated using regression analysis to determine if the equipment reliability has changed over time. If there is a significant trend, a subroutine will be used to obtain the correct values to enter into the Poisson model.

e. Quality of the data

AMMIS data can be used with confidence for the purpose described in this document. This is especially true because of the nature of the data - either the equipment is deemed to have failed or it is not. Also, quality control checks are performed to ensure against coding errors. To eliminate any errors that survived quality control checks, the data were reviewed for consistency and completeness. Some errors were easy to correct using other information recorded on the maintenance action form. However, a few errors could not be corrected because of insufficient information.

f. Inspection of the data

The need to inspect the data for trends, outliers, missing values, etc., before using it in computations was demonstrated with CP140 Aurora equipment replacement data. This work will be done by DAS Eng 5.

g. Errors in the request for data

Often requests for data contain incorrect Work Unit Codes (WUCs). Therefore, the WUCs must be checked against the master list of WUCs known as the Equipment Codes and Inspection Requirements (ECIR) document. To facilitate this checking, each equipment should be described by WUC, part number and nomenclature.

ANNEX C
to ORAE Report No. R87
dated December 1982

PROPOSED USER'S INSTRUCTIONS
CALCULATION OF SPARES REQUIRED TO SUPPORT AIRCRAFT DEPLOYMENTS

CONTENTS

PAGE

PART ONE - GENERAL

Purpose..... C-3
The Basic Concept..... C-3
How This Approach Can Benefit Your Unit..... C-3
How To Contact Us..... C-3

PART TWO - REQUEST FOR DATA

How To Request Data..... C-5
Comments About The Work Unit Code..... C-6
Help In Choosing Items For The Pack-Up Kit... C-7
Time Required To Obtain The Data..... C-7
How Frequently Should You Update The Data.... C-7

PART THREE - HOW TO USE THE DATA

How To Complete The Worksheet..... C-8
Which Nomograph To Use..... C-12
How To Use The Nomographs..... C-12
Revising The Calculations..... C-13
Reliability Trends..... C-15
Component Replacements..... C-15
Scheduled Removals..... C-15
Applying Common Sense..... C-15

Appendix 1 - Sample Print-Out of The Report..... C-17
Appendix 2 - The Nomographs..... C-19

PART ONE - GENERAL

Purpose

1. The purpose of this set of instructions is to enable aircraft maintenance personnel to calculate the number of spares required to support deployments.

The Basic Concept

2. A unit planning an aircraft deployment may send a message to NDHQ requesting failure data on items in its pack-up kit. These data will then be obtained from the Aircraft Maintenance Management Information Systems (AMMIS) and will be forwarded to the unit. The unit can use these data to calculate the number of spares required to provide a desired level of assurance that the pack-up kit will contain enough spares to meet the unit's needs during the deployment. Graphs will be provided which make the computation easy.

How This Approach Can Benefit Your Unit

3. Experience and judgement have traditionally been used to determine the quantity of each spare to include in a pack-up kit. If actual failure data are also used, better results may be obtained.

How To Contact Us

4. Requests or questions regarding data can be forwarded to DAS Eng 5-4-3 in NDHQ:

by message: LOGCON OTTAWA//DAS Eng 5-4-3//

by letter: National Defence Headquarters
Ottawa, Ontario
K1A 0K2
Attention: DAS Eng 5-4-3

by telephone: DAS Eng 5-4-3
AMMIS Development and Analysis
993-0679

5. Comments regarding this report - how it could be improved, how it has benefited your unit, etc. should be forwarded to:

Operational Research and Analysis Establishment
Department of National Defence
Ottawa, Ontario
K1A 0K2
Attention: D Log A

PART TWO - REQUEST FOR DATA

How To Request Data

6. To obtain failure data, an unclassified, routine message should be sent to:

LOGCON OTTAWA//DAS ENG 5-4-3//

The subject of the message should be:

SUBJ: FAILURE DATA, PACK-UP KIT ITEMS

The message must contain information as follows:

a. For all aircraft except CF101, CF104, CF116 and CC130:
   (1) type of aircraft to be deployed;
   (2) total number of hours to be flown during the deployment; and
   (3) list of spares to be included in the pack-up kit identified by the Work Unit Code (WUC), part number, and nomenclature.

Example

SUBJ: FAILURE DATA, PACK-UP KIT ITEMS

1. CC115-BUFFALO
2. 200 FLYING HOURS
3.

WUC	PART NUMBER	NOMENCLATURE
AHDBA	8.90 X 12.50	NOSE TIRE
BCUF	108458-2-1	VALVE REGULATOR
DABA	321774-2-1	VALVE HOT AIR MIXING
etc.		

b. For aircraft with more than one configuration, namely CF101, CF104, CF116 and CC130:

(1) each type of aircraft to be deployed;
(2) total number of hours to be flown by each aircraft type during the deployment; and
(3) list of spares to be included in the pack-up kit identified by the Work Unit Code (WUC), part number nomenclature.

Example

SUBJ: FAILURE DATA, PACK-UP KIT ITEMS
1. A. CF116A - FREEDOM FIGHTER
   B. CF116D
2. A. 300 FLYING HOURS
   B. 200 FLYING HOURS
3.

WUC	PART NUMBER	NOMENCLATURE
ABBF	3-1350 5-1	CANOPY BALANCE CABLE
BEJE	12A00064P01	FLAME HOLDER
GATTA	8010000083-12	TACAN ARN 504
etc.		

If your deployment consists of more than one aircraft type, for example CC130s and CC115s, then a separate message is required for each aircraft type.

Comments About The Work Unit Code

7. It is important to ensure that the WUCs are correct, otherwise the wrong data may be provided or delays in providing the data may occur.

8. Be especially careful regarding items that are installed in the aircraft in more than one location (ie, upper, lower or right, left). Such items may have a different WUC for each location. Include all WUCs for these particular items.

Help In Choosing Items For The Pack-Up Kit

9. Items included in a pack-up kit are usually those that were required during previous deployments. If this is the first deployment, if records of past usage have not been kept, or if you suspect that the reliability of some equipment has changed, you may need help in choosing which items to include in the pack-up kit. If so, contact DAS Eng 5-4-3 by telephone. A computer sort of failure data can be performed and a print-out listing items in order of frequency of failure will be sent to you.

Time Required To Obtain The Data

10. A request for data will be actioned as quickly as possible. However, time must be allowed for the message to reach DAS Eng 5-4-3, the computer program to be run, the results to be checked and the print-out to be mailed to the unit. This will take at least three weeks.

How Frequently Should You Update The Data?

11. Since equipment failure rates do not change quickly, the failure data should be suitable for planning aircraft deployments for at least one year. However, the pack-up kit contents should be reviewed before each deployment. Also, hours flown and spares used should be recorded after each deployment. If there is concern regarding any items, data can be obtained from DAS Eng 5-4-3.

PART THREE- HOW TO USE THE DATA

How To Complete The Worksheet

12. The terms that you will need to know are defined in Table C-1. The data that you will receive from DAS Eng 5-4-3 will be printed on worksheets illustrated by Figure C-1. An example print-out is given in Appendix 1.

13. The only part of the Worksheet that you will have to complete is the part entitled "DEPLOYMENT PLANNING". Instructions are provided below regarding each entry.

14. T_D - DAS Eng 5-4-3 will sample several years of AMMIS data by dividing the total time period into intervals of equal flying hours duration, t_s. The number of equipment replacements occurring during each interval will then be determined. The duration, T_D, of the planned deployment may differ from t_s and, therefore, should be recorded on the worksheet. For example, the data may have been obtained using a sample interval of t_s = 300 hours. However, the planned duration of the deployment may be T_D = 400 hours.

15. T_D/t_s is a comparison of the duration of the deployment with the duration of the interval used for data sampling. For example, if T_D = 400 but t_s= 300, T_D/t_s = 400/300 = 1.33.

16. M - The mean number of replacements of a particular item expected during the deployment is calculated as follows:

$$M = m \cdot T_D/t_s$$

For example, if m = 0.8 and T_D/t_s = 1.33 then:

$$M = (0.8)(1.33) = 1.064$$

TABLE C-1

TERMS USED IN COMPUTATIONS

EQUIPMENT DESCRIPTION

Symbol	Term	Meaning
WUC	–	Work Unit Code
–	Part No. & Nomenclature	A brief description of the item

AMMIS DATA

t_s	Time interval used in sampling	The duration of the time interval used to sample AMMIS data expressed in flying hours.
m	Mean	The mean (average) number of replacements of a particular item per sample interval.
v	Variance	The variance in the number of replacements of a particular item per sample interval - a measure of how widely these numbers vary from the mean.
R	Ratio	The ratio v/m.

DEPLOYMENT PLANNING

T_D	Deployment Duration	The planned duration of the deployment expressed in flying hours.
T_D/t_s	-	A comparison of the duration of the deployment with the duration of the interval used for data sampling.
M	Mean	The mean number of replacements of a particular item expected during the deployment.
P	Probability	The probability (likelihood) of having a spare unit available when required, expressed as a number between zero (no chance) and one (certainty).
S	Spares	The number of spares of a particular item included in the pack-up kit.

WORKSHEET

AIRCRAFT TYPE_____ DATE____

EQUIPMENT DESCRIPTION			AMMIS DATA t_s= _____FLYING HOURS				DEPLOYMENT PLANNING T_D = ____FLYING HOURS			
NO.	WUC	PART NO & NOMENCLATURE	m	v	R	NOMOGRAPH	T_D/t_s	M	P	S
1	XXXX	XXXXXXXX	XXXX	XXXX	XXX	X	___	___	___	___
2	XXXX	XXXXXXXX	XXXX	XXXX	XXX	X	___	___	___	___
3	XXXX	XXXXXXXX	XXXX	XXXX	XXX	X	___	___	___	___
4	XXXX	XXXXXXXX	XXXX	XXXX	XXX	X	___	___	___	___
etc.										

Figure C-1: Worksheet

17. P - This is the probability of having a spare unit available when required. It can be thought of as the percentage of a large number of similar deployments that could be completed without running short of that item. For example, if P = 0.95, then one would expect that there would be enough spares in 95 out of 100 deployments. P is a goal or target that must be chosen by those responsible for planning the deployment. There are two possible approaches:

a. Treat all items the same - choose a value of P for the kit as a whole, say 0.80 and then determine what value is needed for each individual item. For a kit containing 20 items, the solution obtained by using a hand held calculator is $(0.80)^{1/20} = 0.9889$ or approximately 0.99 for each item.

b. Treat the items differently - assign different P values to each of the individual items, if you consider some items to be more critical than others. For example, you may want to assign a value of 0.95 for a radio and 0.98 for an engine component. All the values could then be multiplied together to determine the value of P for the kit as a whole. For example, consider a kit containing four items: $(0.95)(0.98)(0.995)(0.90) = 0.83$.

Note that the value chosen for P should be fairly high - at least 0.85.

18. S - Given values for M and P, the number of spares required can be calculated using the nomographs provided. Alternatively, if M and S are known, P can be determined.

19. An allowance is already made in the equipment replacement data for the quantity of each item installed in the aircraft. However, for some aircraft types (CF101, CF104, CF116 and CC130) different models exist and consequently, each model may have a different quantity of an item installed. The program which extracts equipment data will provide separate data for each model.

Which Nomograph To Use

20. Four different nomographs are provided in Appendix 2. They were developed for the values R=1, R=2, R=3 and R=5. However, they can be used for other values of R as indicated below (with R rounded to nearest decimal):

Nomograph	Use When R is Between
A	0 and 1.4
B	1.5 and 2.4
C	2.5 and 3.4
D	above 3.4

How To Use The Nomographs

21. The number of spares, S, can be computed using the following steps:

a. calculate the value of M as shown in paragraph 16;

b. choose a value for P;

c. select the appropriate nomograph;

d. use a transparent straight edge to join the value of M to the value of P; and

e. read the answer off the curve for S.

22. For example, say R = 0.9, M = 0.8, P = 0.98. How many spares are required?

23. Nomograph 'A' should be used since R is less than 1.5. As shown in Figure C-2, line 1 joins M=0.8 to P=0.98. Line #1 intersects the spares curve at 2.7. However, this value should be rounded up to the next whole number, i.e. S=3.

24. Note that taking three spares actually results in P = 0.991. This is found by joining M=0.8 to S=3 and reading the value of P as shown in Figure C-2, line #2.

25. But what if three spares are not available? Say only one may be included in the pack-up kit. Joining M=0.8 to S=1 gives P=0.80 (see figure C-2, line #3). Thus, reducing the number of spares from 3 to 1 decreases the probability of having enough by 0.99 - 0.80 = 0.19. In other words, the chance of a stock-out has increased by 19%.

Revising The Calculations

26. The calculations easily can be revised for the next deployment. If the type of aircraft to be deployed is different, check to see if the number of equipment installations is affected. If so, M will have to be recalculated using new data. Also M should be recalculated if the duration of the deployment, T_D, were significantly different from t_s.

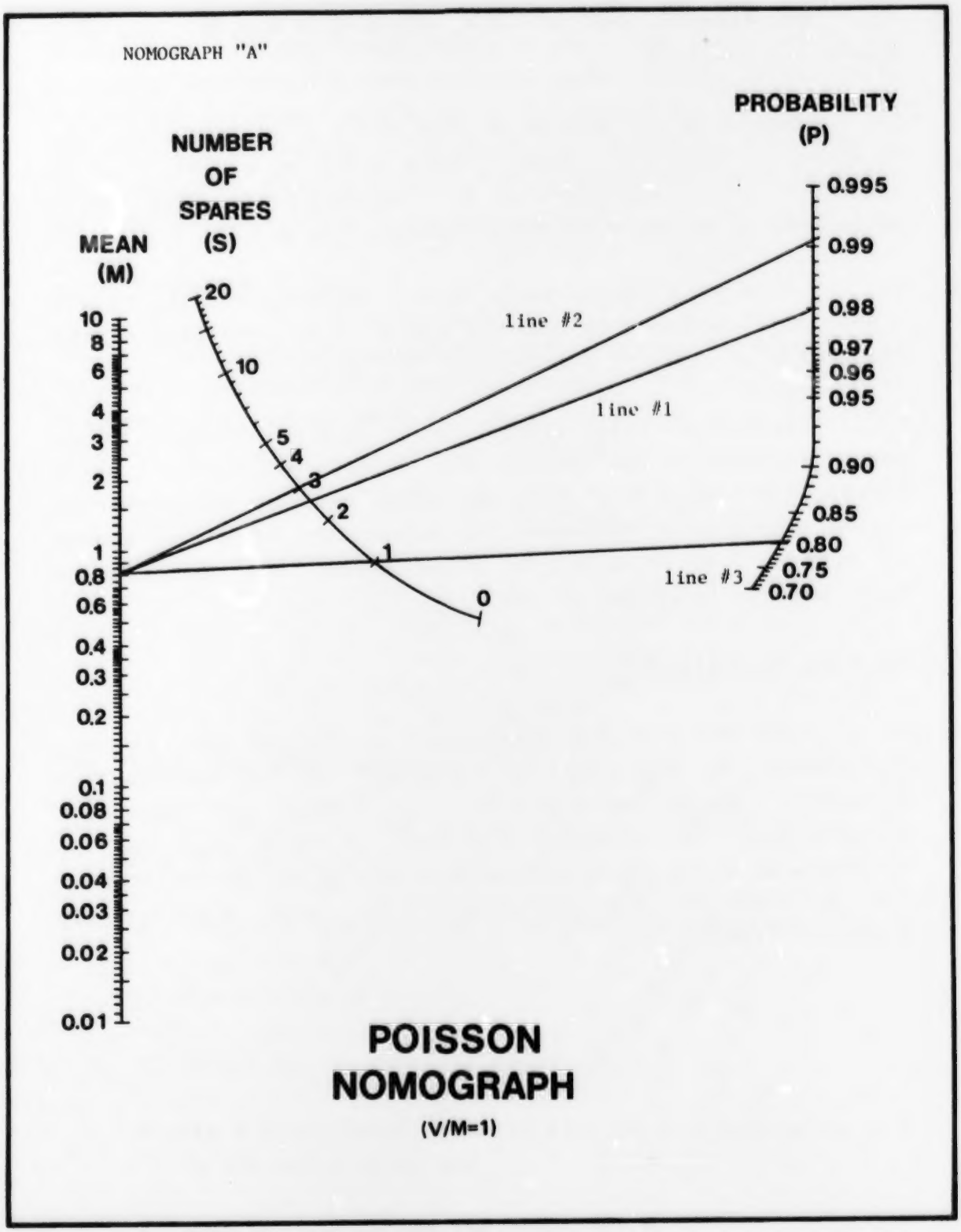


Figure C-2: Examples

Reliability Trends

27. The computer program used by DAS Eng 5-4-3 to extract data from AMMIS produces graphs that can be used to detect trends in equipment reliability. Because of the possibility of trends, the data should be updated annually.

Component Replacements

28. The data provided by DAS Eng 5-4-3 is based on the assumption that during the deployment, repair will be by equipment replacement and that no component spares will be available. If component spares are included in the pack-up kit, data can be obtained regarding their consumption rate.

Scheduled Removals

29. Additional spares should be included in the pack-up kit if scheduled removals will occur during the deployment.

Applying Common Sense

30. This approach is not intended to replace experience or common sense, but to augment it with data and techniques not previously available to deployment planners. Therefore, after you have done the calculations, ask yourself:

"Do the answers make sense?"

If not, look for a possible source of error. For example, consider the Astral Compass used in the Sea King Helicopter. It is only installed during northern deployments, but the AMMIS failure data would reflect failures as if the compass were

always installed. Thus, the failure rate estimates based on AMMIS data would be too low. It is not unusual for equipment to be used intensively during deployments but only occasionally otherwise. This sort of situation should be brought to the attention of DAS Eng 5-4-3.

31. The techniques described above should provide good results. Canadian Forces units are encouraged to use them. Also, units are encouraged to keep records of their deloyments. Figure C-1 could be amended for this purpose by adding the entry " _____ FLYING HOURS (ACTUAL)" and a column for "SPARES USED". Such records could be used to evaluate and improve the deployment planning process.

SAMPLE PRINT-OUT OF THE REPORT

CALCULATION OF SPARES REQUIRED FOR AIRCRAFT DEPLOYMENT
WORKSHEET

AIRCRAFT TYPE: 116 AIRCRAFT MARK(S): BD --
RUN DATE: DECEMBER 16, 1982 HISTORY START DATE: OCT 1975 HISTORY STOP DATE: SEP 1981
PAGE: 1

EQUIPMENT DESCRIPTION			AMMIS DATA T_S = 450 FLYING HOURS				DEPLOYMENT PLANNING T_D = FLYING HOURS			
NO.	WUC (OR GROUPED WUCS)(1ST 5 WUCS ONLY)	NOMENCLATURE	M*	V*	R*	NOMOGRAPH	T_D/T_S	M	P	S
1	ABBF---	CABLE ASSY-CBAL	1.092	2.343	2.15	B				
2	AHAA---	BRAKE ASSY	0.798	1.218	1.53	B				
3	AHBE---	ACTUATOR ASSY	0.303	0.435	1.44	A				
4	AHGL---	LINE ASSY-MLG	0.229	0.197	0.86	A				
5	BEJE---	FLAMEHOLDER-AB	0.734	0.956	1.30	A				
6	BEKA--- BEKC--- BEKD---	LINER-AB REAR	0.128	0.187	1.46	A				
7	BEKH---	LEAF ASSY OUTER	0.514	1.382	2.69	C				
8	BER----	ACCESSORY DRIVE	2.422	5.339	2.20	B				
9	BESA---	POWER UNIT-VEH	0.881	1.587	1.80	B				
10	BEYK---	PLUG-IGNITER AB	1.055	1.904	1.80	B				
11	DAEd---	SEAL ASSY-WSHLD PNEJ	0.183	0.170	0.92	A				
12	DAFA---	DUCT ASSY	0.028	0.027	0.98	A				
13	DAFB---	DUCT ASSY-CAN DEFOG	0.101	0.147	1.46	A				
14	DAFC---	SEAL-CANOPY DEFOG	0.000	0.000	0.00	A				
15	DAFD---	SELECTOR-DEFOG CONT	0.009	0.009	1.00	A				
16	DAGA---	VALVE-ANTI-G	0.248	0.225	0.91	A				
17	DAGB---	GOISC-SEAT&ANTI-G	0.037	0.036	0.97	A				
18	DAHC---	BLOWER-COMPT COOLING	0.257	0.267	1.04	A				
19	DAHD---	BLOWER-COMPT COOLING	0.092	0.103	1.12	A				
20	DAHE---	ACTR-DOOR COOLING	0.138	0.175	1.27	A				

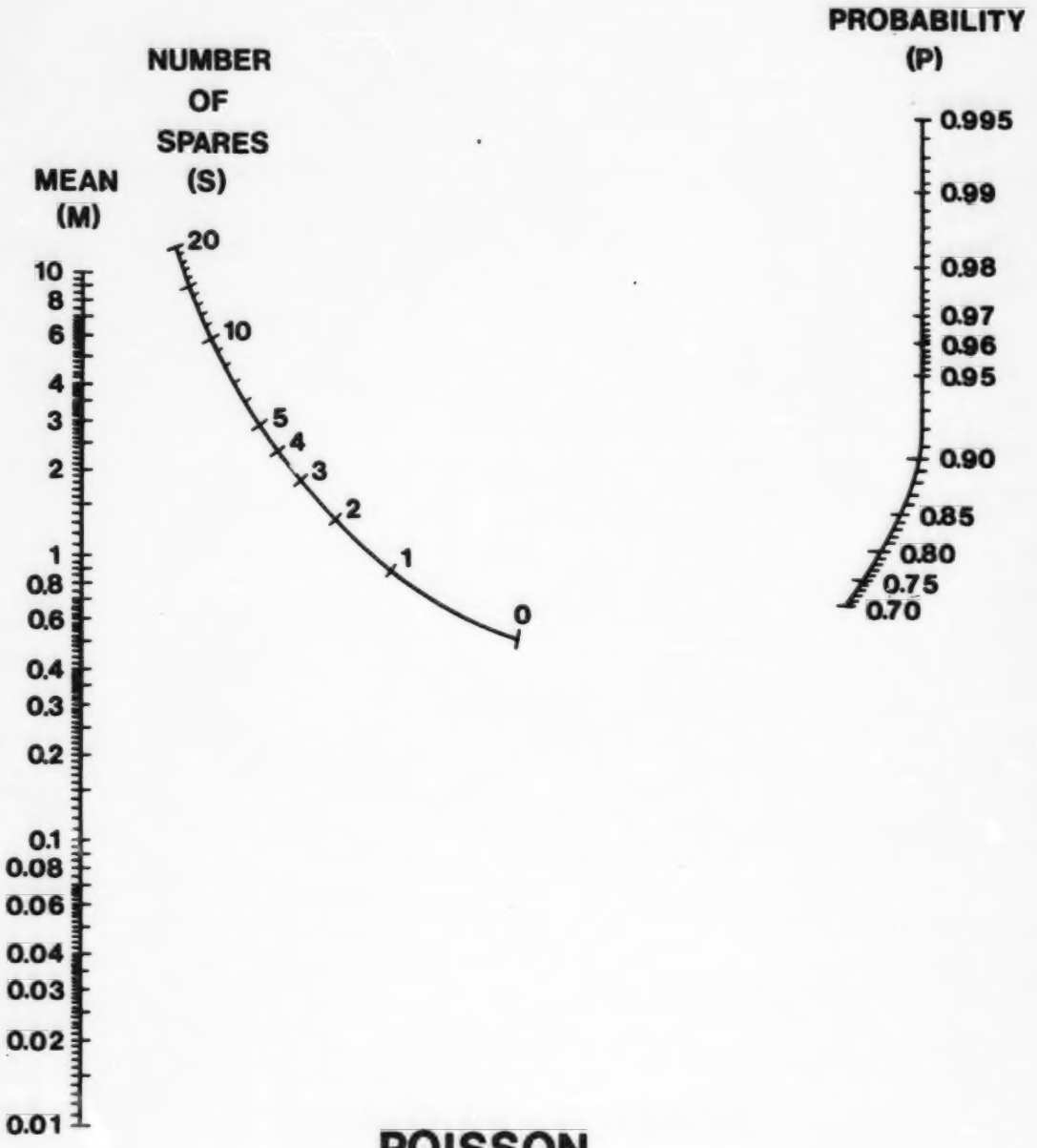
'*' DENOTES THAT THESE CAPITAL LETTERS SHOULD BE CONSIDERED AS LOWER CASE

NOTE: This listing is to be used for illustrative purposes only.

APPENDIX 2

THE NOMOGRAPHS

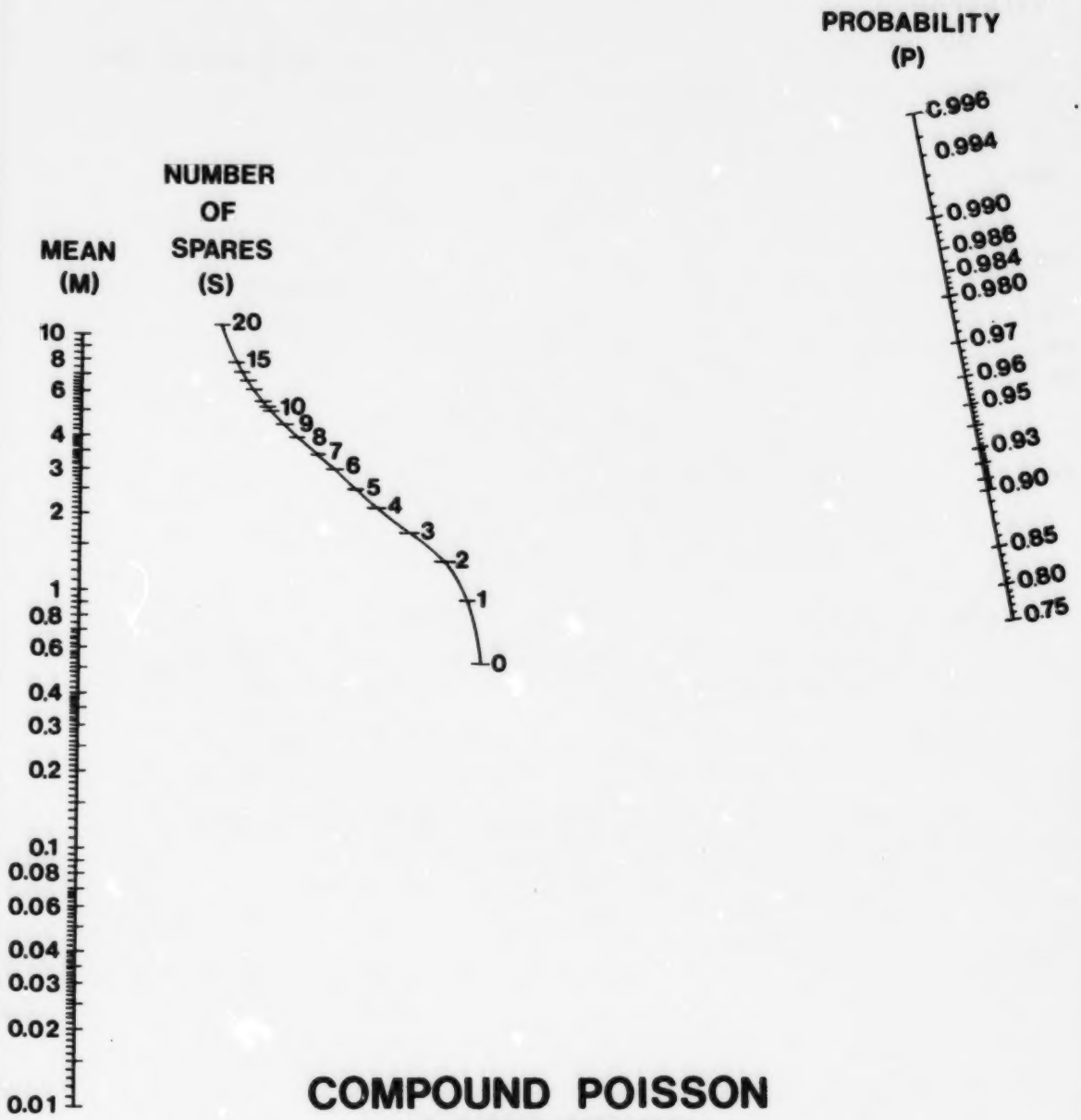
NOMOGRAPH "A"



POISSON NOMOGRAPH

(V/M=1)

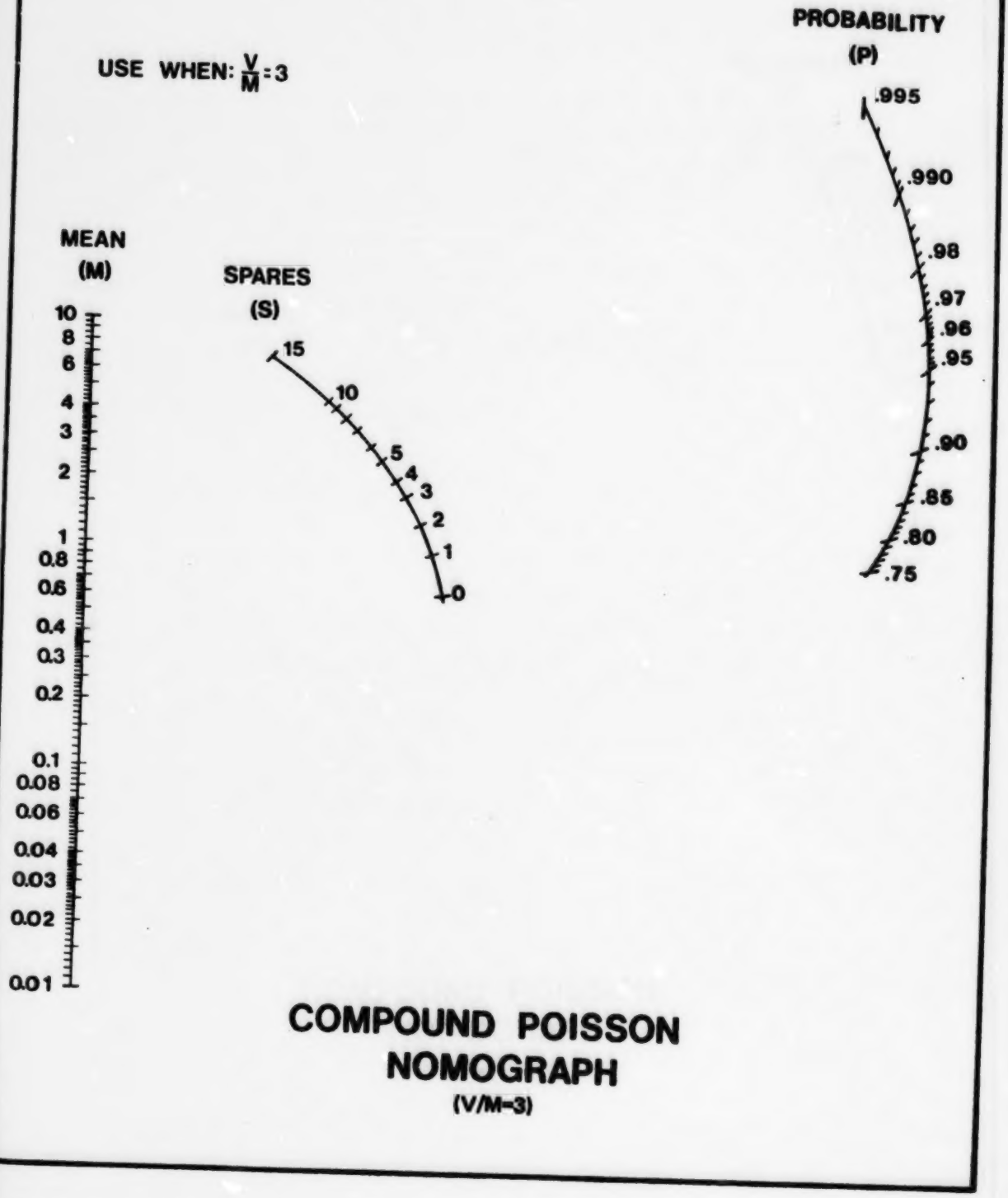
NOMOGRAPH "B"



COMPOUND POISSON NOMOGRAPH

(V/M=2)

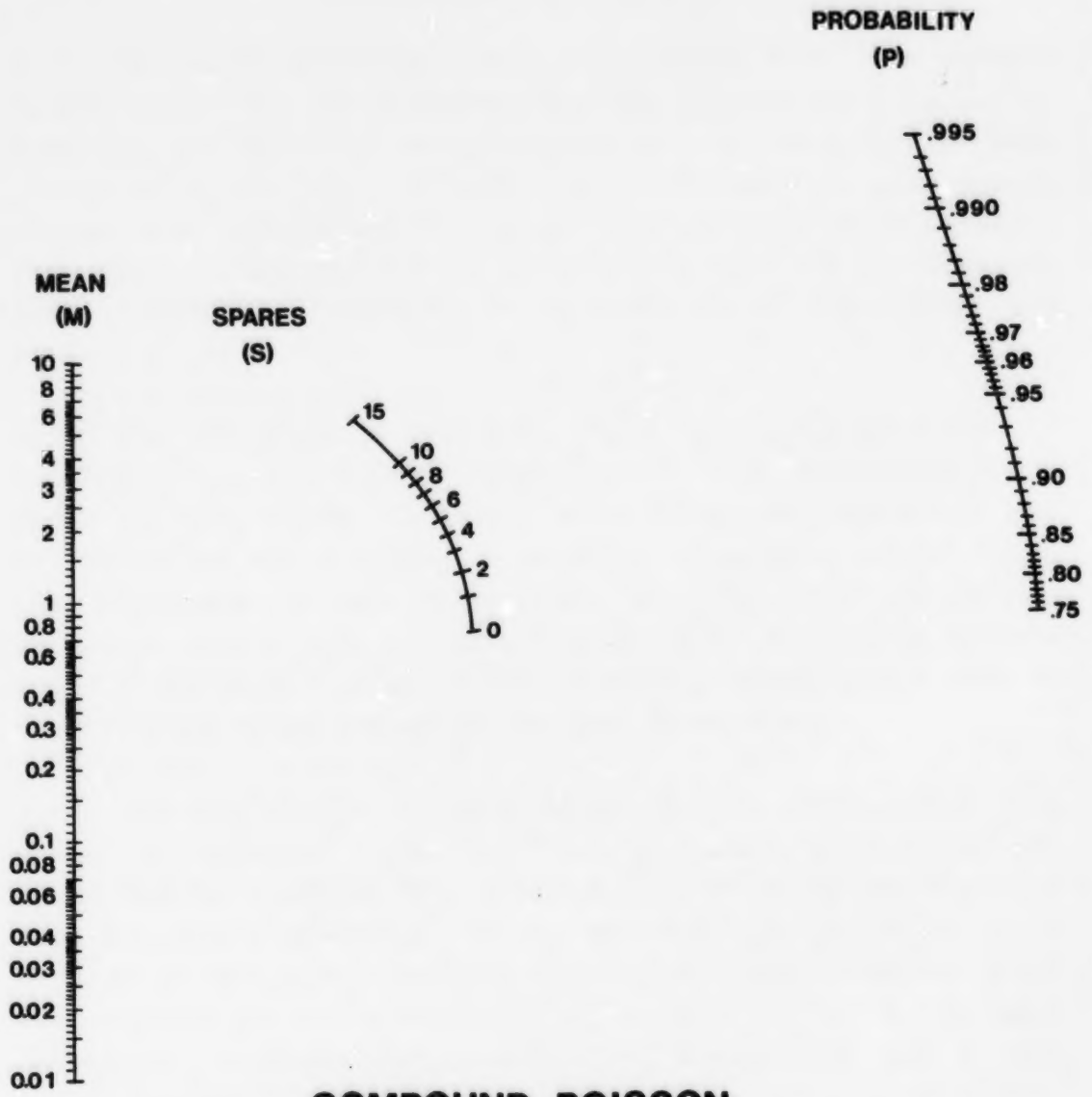
NOMOGRAPH "C"



COMPOUND POISSON NOMOGRAPH

(V/M=3)

NOMOGRAPH "D"



COMPOUND POISSON NOMOGRAPH

(V/M = 5)

ANNEX D
to ORAE Report No. R87
dated December 1982

CREATION OF THE NOMOGRAPHS

1. Having obtained equipment replacement data, the authors became aware that the nomograph for the Poisson model (given in Reference B), which is characterized by a variance (V) to mean (M) ratio of one, was not sufficient. Variance to mean ratios of two were common and ratios as high as five were evident. Therefore, it was decided to create nomographs for the Compound Poisson model for variance to mean ratios of two, three and five.

2. The nomograph in Reference B was originally published in Reference C. The latter document has been superseded by a later edition which contains no information regarding how the nomograph was created. A thorough literature search, with the assistance of the Directorate of Scientific Information Services (DSIS), also failed to reveal any information on this point. Therefore, it was realized that a method would have to be devised to create Compound Poisson nomographs.

3. The key concept required to solve this problem came from elementary geometry - the fact that simultaneous equations can be solved by locating the intersection of lines representing each respective equation. It was realized that this idea could be used to calibrate new sets of curves. For example, given two properly located and calibrated curves, say for M (the mean number of equipment replacements per deployment) and P (the target probability of having enough spares of a given item), one could locate and calibrate the curve for S (the required number of spares). This point is illustrated by Figure D-1, in

which an attempt is made to locate a point for the S-curve, namely S = 4, for the case V/M = 3, given the original curves for M and P.

4. In order to plot the required lines, a table of values for M, S and P was needed. A computer program was written for this purpose, using the Negative Binomial form of the Compound Poisson model.

5. Unfortunately, as illustrated by Figure D-2, plotting this data using the original location and calibration for the M and P-curves did not yield a point for the S-curve. The lines did not meet at a focal point, rather they yielded a locus that traced out a series of locations along a smooth curve for a given value of S. The same applied for all values of S. This was a serious problem because an accurate value for P could not be obtained given values for M and S.

6. To solve this problem, it was decided to leave the location and calibration of the M-curve fixed, making it a 'known' element. Then the P-curve could be moved to a better location and a new S-curve plotted. Various schemes were tried to project the calibration from the original P-curve to the P-curve at the new location.

7. The next refinement was the realization that given a fixed M-curve, and any two points on the S-curve (say 0 and 1), one could obtain a good location and calibration for the P-curve. The two points on the S curve could be located arbitrarily, albeit conveniently. This is illustrated in Figure D-3.

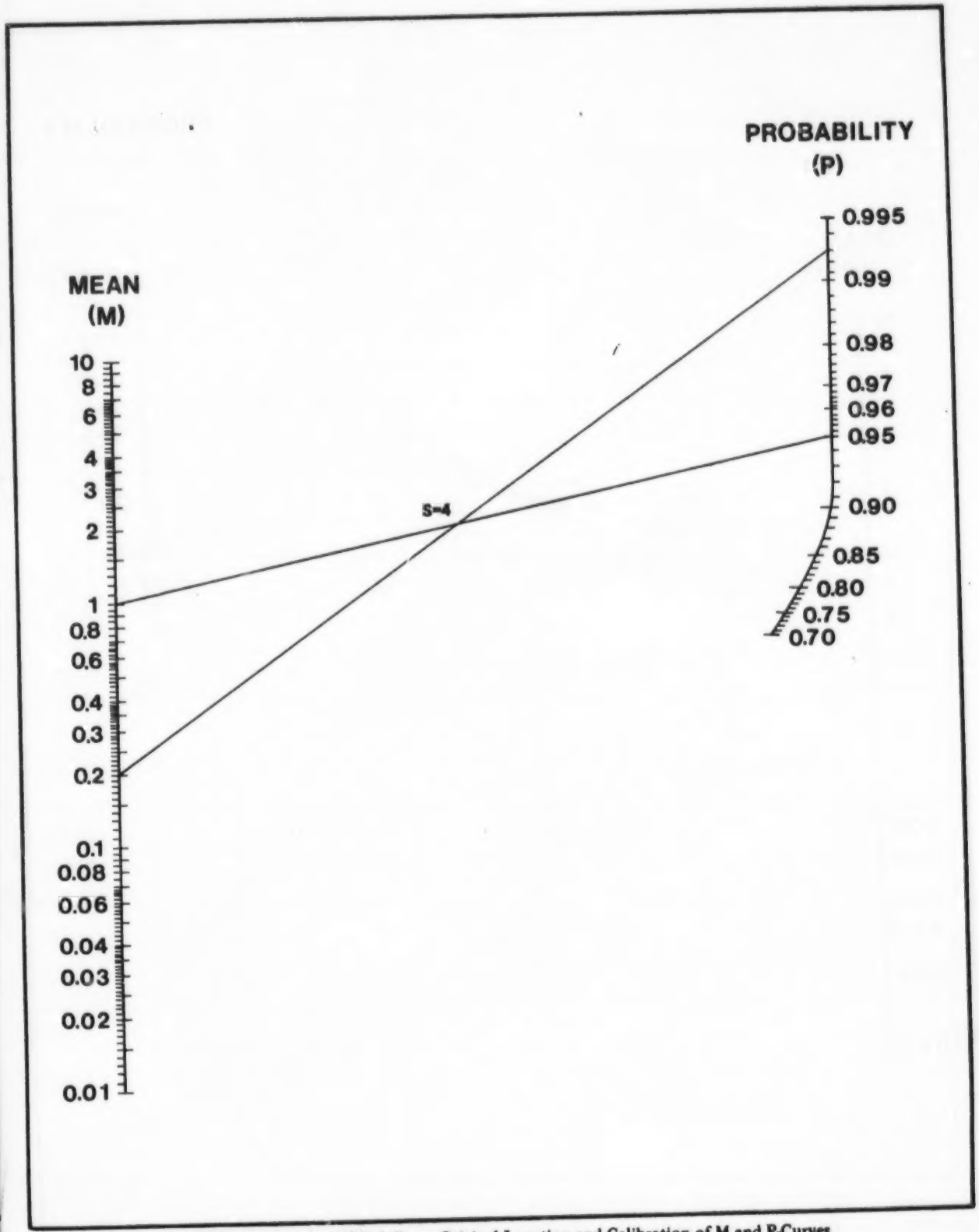


Figure D-1: Attempt to Locate S=4 for V/M=3 Given Original Location and Calibration of M and P-Curves

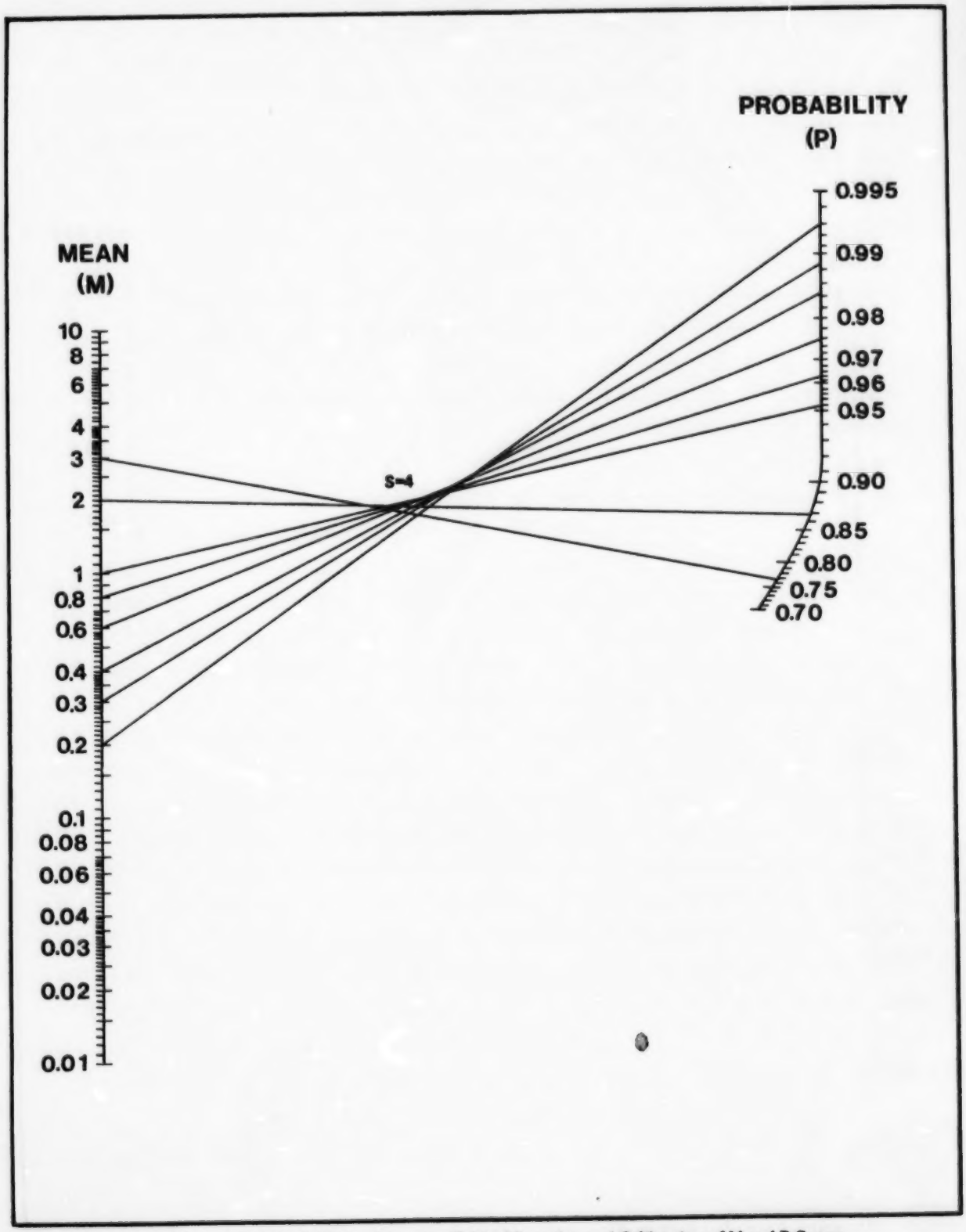


Figure D-2: Lack of Focus at S=4 for V/M=3, when using Original Location and Calibration of M and P-Curves

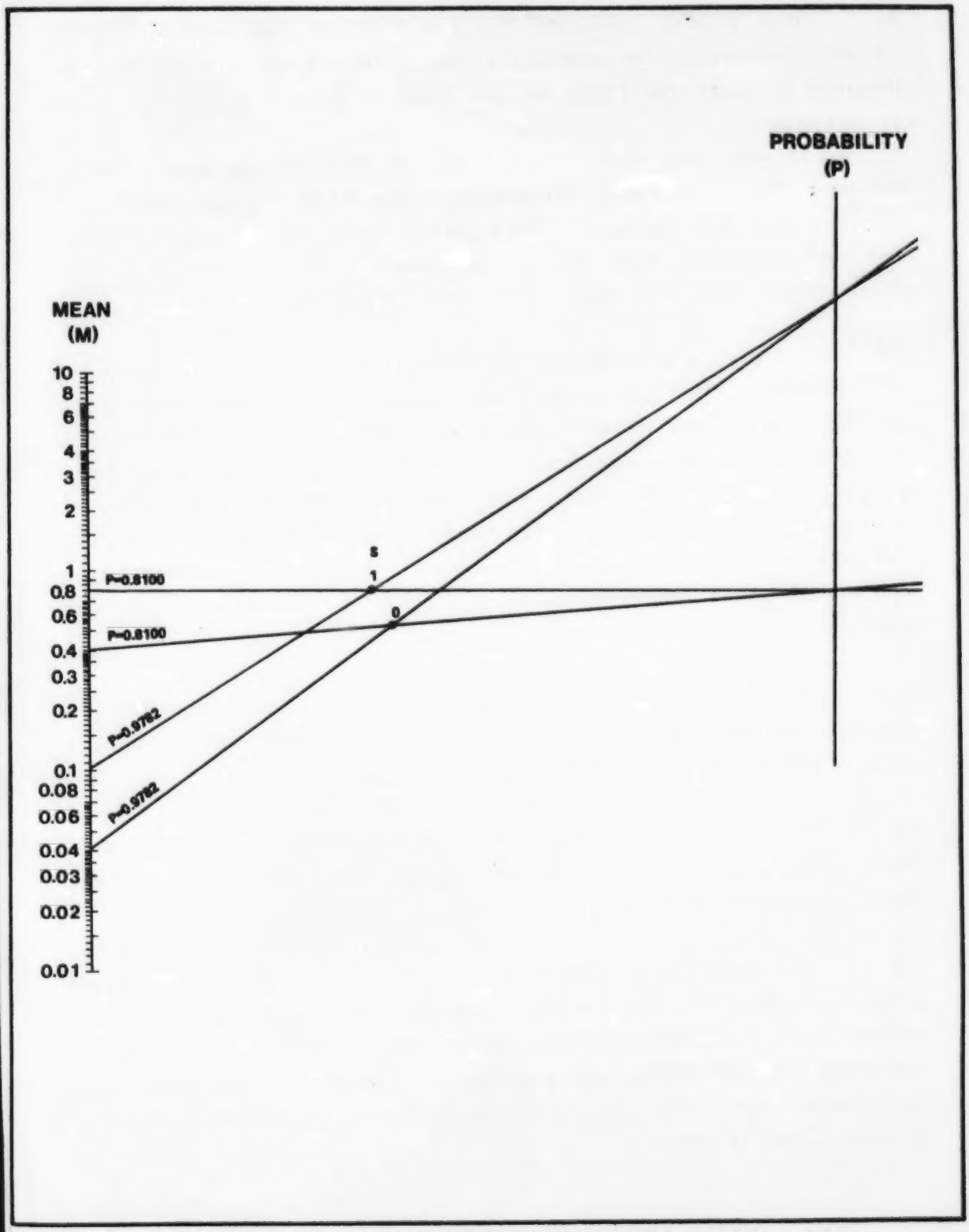


Figure D-3: Locating and Calibrating P-Curve, given two Arbitrarily but Conveniently Located Points on S-Curve

8. Once a good location for the P-curve had been obtained, it was calibrated by drawing lines from various points on the M-curve through one point on the S-curve (say S = 0).

9. It was then discovered that the P-curve is a compromise between solutions that correspond for high values of P but diverge for low values. This point is illustrated in Figure D-4 and explains why, in the original Poisson nomograph, the P-curve was bent and why it did not extend below P=0.70.

10. Finally, a systematic (although tedious) method was used to achieve the necessary accuracy. The location and calibration of the M-curve was fixed, and initial solutions for S and P-curves were obtained. Then the P-curve was considered fixed and the location and calibration of the S-curve improved. Next the S-curve was considered fixed and the calibration of the P-curve improved. This process was continued until convergence on a highly accurate solution was obtained.

11. One method used to fine tune the calibration of the P-curve was to bend it slightly - this explains its curvature in the solutions for V/M = 3 and V/M = 5.

12. The hand-drawn nomograph was expressed in publishable form by graphic artists. Retaining accuracy of calibration was difficult at this stage.

13. The resulting nomographs reflect, to some degree, a learning process. The curves might have had slightly different shapes had the procedure been fully developed before the first Compound Poisson nomograph was drawn. However, each nomograph has been carefully checked for accuracy against computer generated solutions.

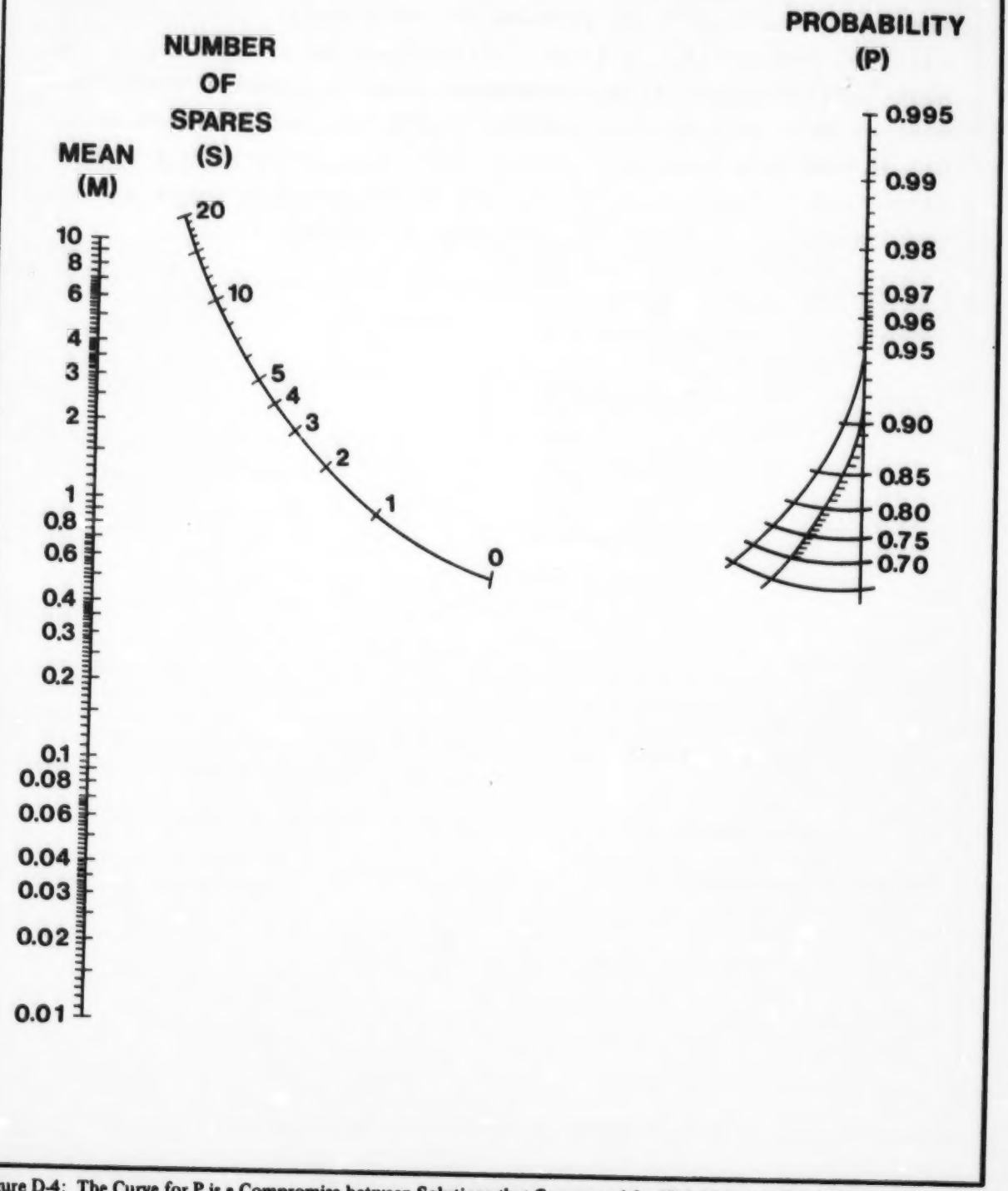


Figure D-4: The Curve for P is a Compromise between Solutions that Correspond for High Values of P but Diverge for Low Values. (The Drawing is Exaggerated to Illustrate the Point.)

14. Should any reader wish to produce nomographs, some advice on materials is warranted. It is best to work on a large scale and then photographically reduce the final product. A light table is essential - it is made of translucent glass brightly lit from underneath. A tough, transparent paper is needed, the most suitable is called 'Albanene' tracing paper. Also, one must be able to draw very precise lines. A good approach is to use an ordinary hard lead pencil kept very sharp with a pad of fine sanding paper. Finally, one needs a human being with a combination of imagination, optimism and tenacity.

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13. ABSTRACT

This report provides graphical methods that can be used by aircraft maintenance personnel to calculate the number of spares needed to support a deployment. It explains the theory involved, presents illustrative computations and shows how the required data can be obtained from the Aircraft Maintenance Management Information System (AMMIS). A user's manual is included to facilitate implementation.

The concepts are applicable to any equipment maintenance situation where repair is by replacement and repair of failed items is deferred. The concepts could be implemented in the air, land, sea and communications and electronics environments.

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73-159

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Security Classification

KEY WORDS

Aircraft deployments
Repair by replacement
Spare parts required
Poisson distribution
Negative binomial
Nomographs (nomograms)

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